Comparison of different time splitting algorithms for positive definite advection

Werner Schneider, Andreas Bott

Meteorological Institute, University of Bonn

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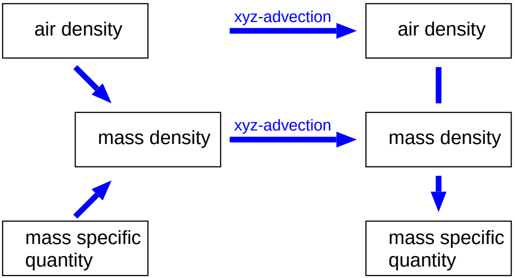
Motivation

advection schemes in the COSMO model

- 1d advection
  $\rightarrow$ time splitting algorithm required (e.g. Strang splitting)
- flux form advection
  $\rightarrow$ volume specific quantities needed
  $\rightarrow$ density advection required
- time splitting errors destroy consistency
  $\rightarrow$ density advection required

Motivation

outline of an advection step



but . . .

- density may become zero
  → stability problems
- mass change due to diagnostic density
- consistency for strong gradients?

The time splitting problem

for simplicity: upstream advection, Courant number 0.5

density

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

scalar [**/kg]

1	1	1	1
1	1	1	1
1	1	2	1
1	1	1	1

The time splitting problem

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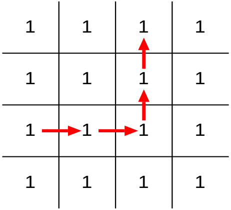
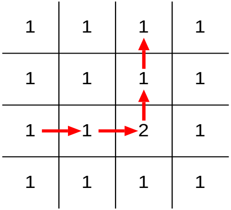
density



scalar [**/m³]



The time splitting problem

for simplicity: upstream advection, Courant number 0.5

y-x advection

density

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

density

		1	
		1	
1	1	1	

scalar [**/m³]

1	1	1	1
1	1	1	1
1	1	2	1
1	1	1	1

scalar [**/m³]

		1	
		1.5	
1	1	1.5	

The time splitting problem

for simplicity: upstream advection, Courant number 0.5

y-x advection

density

1	1	1	1
1	1	1	1
1	1	1	1
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density

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1	1	1	1

scalar [**/m³]

1	1	1	1
1	1	1	1
1	1	2	1
1	1	1	1

scalar [**/kg]

		1	
		1.5	
1	1	1.5	

The time splitting problem

for simplicity: upstream advection, Courant number 0.5

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1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

scalar [**/m³]

1	1	1	1
1	1	1	1
1	1	2	1
1	1	1	1

y-x advection

density

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

scalar [**/kg]

		1	
		1.5	
1	1	1.5	

x-y advection

density

		1	
		1.25	
1	1	0.75	

scalar [**/m³]

		1	
		1.75	
1	1	1.25	

The time splitting problem

for simplicity: upstream advection, Courant number 0.5

density

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

scalar [**/m³]

1	1	1	1
1	1	1	1
1	1	2	1
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y-x advection

density

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

scalar [**/kg]

		1	
		1.5	
1	1	1.5	

x-y advection

density

		1	
		1.25	
1	1	0.75	

scalar [**/kg]

		1	
		1.4	
1	1	1.67	

The time splitting problem

for simplicity: upstream advection, Courant number 0.5

density

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

y-x advection

density

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

x-y advection

density

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

scalar [**/m³]

1	1	1	1
1	1	1	1
1	1	2	1
1	1	1	1

scalar [**/kg]

		1	
		1.5	
1	1	1.5	

scalar [**/kg]

		1	
		1.4	
1	1	1.67	

mass gain 0.07

Deformational correction algorithm

transport equation for substance with mass density ψ:

$$\frac{\partial \psi}{\partial t} = -\nabla \cdot (\psi \mathbf{v}) = -\sum_{i=1}^{3} \frac{\partial}{\partial x_i} (\psi \dot{x}_i)$$

rewritten as

$$\frac{\partial \psi}{\partial t} = -\sum_{i=1}^{3} \left[\frac{\partial}{\partial x_i} (\psi \dot{x}_i) - \psi \frac{\partial \dot{x}_i}{\partial x_i} \right] - \psi \nabla \cdot \mathbf{v}$$

Deformational correction algorithm

transport equation for substance with mass density ψ:

$$\frac{\partial \psi}{\partial t} = -\nabla \cdot (\psi \mathbf{v}) = -\sum_{i=1}^{3} \frac{\partial}{\partial x_i} (\psi \dot{x}_i)$$

rewritten as

$$\frac{\partial \psi}{\partial t} = -\sum_{i=1}^{3} \left[\frac{\partial}{\partial x_i} (\psi \dot{x}_i) - \psi \frac{\partial \dot{x}_i}{\partial x_i} \right] - \psi \nabla \cdot \mathbf{v}$$

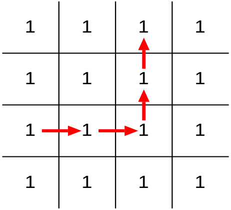
↗ calculated during 1d advection steps

↖ subtracted after 3d advection step

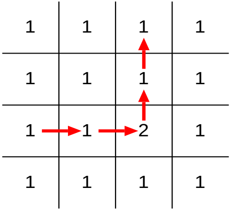
Advection with deformational correction

for simplicity: upstream advection, Courant number 0.5

density



scalar [**/m³]



Advection with deformational correction

for simplicity: upstream advection, Courant number 0.5

y-x advection

density

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1	1	1	1
1	1	1	1
1	1	1	1

density

		1	
		1	
1	1	1	

scalar [**/m³]

1	1	1	1
1	1	1	1
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scalar [**/m³]

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Advection with deformational correction

for simplicity: upstream advection, Courant number 0.5

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density

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1	1	1	1
1	1	1	1
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scalar [**/m³]

1	1	1	1
1	1	1	1
1	1	2	1
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scalar [**/kg]

		1	
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Advection with deformational correction

for simplicity: upstream advection, Courant number 0.5

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scalar [**/m³]

1	1	1	1
1	1	1	1
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y-x advection

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1	1	1	1
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scalar [**/kg]

		1	
		1.5	
1	1	1.5	

x-y advection

density

		1	
		1	
1	1	1	

scalar [**/m³]

		1	
		1.25	
1	1	1.75	

Advection with deformational correction

for simplicity: upstream advection, Courant number 0.5

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scalar [**/m³]

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1	1	1	1
1	1	2	1
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y-x advection

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scalar [**/kg]

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x-y advection

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1	1	1	1

scalar [**/kg]

		1	
		1.25	
1	1	1.75	

Comparative case studies

- time splitting errors
  - both schemes: dependency on time splitting sequence
  - operational scheme: only consistent with density advection
- mass errors due to diagnostic density

$\rightarrow$ Which time splitting algorithm is recommended?

$\rightarrow$ tests with real case studies

Comparative case studies

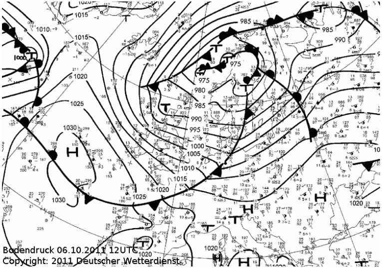
model set up

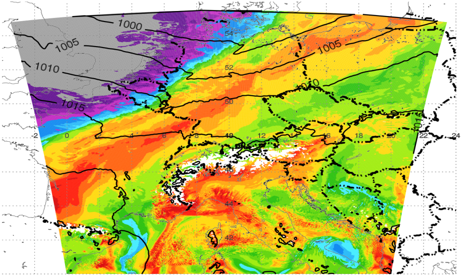
- fourth order Bott scheme
- no divergence damping
- different Strang splitting approaches:
  - operational approach xyz- and zyx-advection alternating (Δt)
  - true Strang splitting zyxyz advection ($\Delta t/2$)
  - double Strang splitting zyxy z yxyz ($\Delta t/4$)
- different resolutions (DE, EU)
- operational (ref) and deformational correction (dc) algorithm
- passive tracer

Weather Situation

requirements:

- strong deformational flow
  → alpine region
  → meteorological fronts
- example October 6, 2011





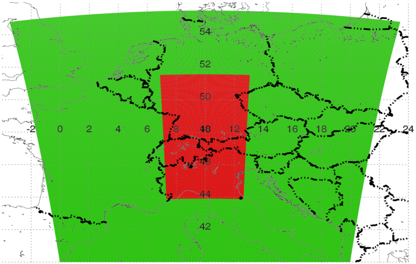
Pseudopotential temperature [°C] 850hPa, Sea level pressure [hPa]

≤20.0 (Δ =2.00) ≥50.0

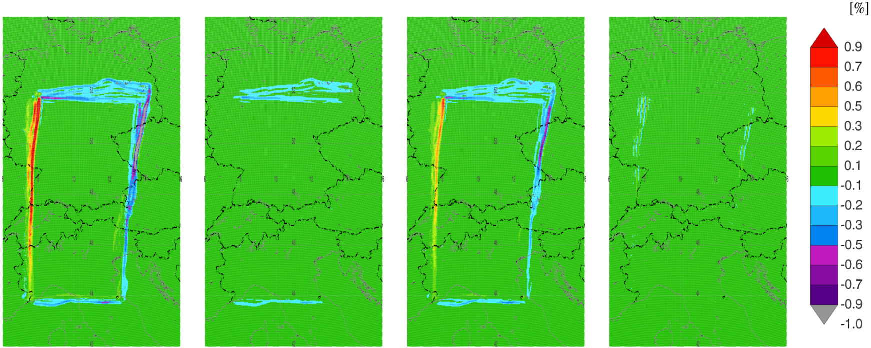
Results

Tracer initialization

- strong gradients
- box centered over alpine region
- pure advection



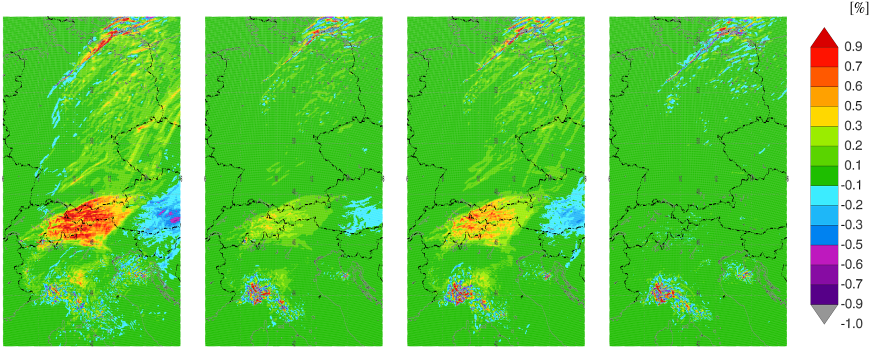
Tracer advection



ref (Δt)- dc (Δt) | ref (Δt) - ref (Δt/2) | ref (Δt)- ref (Δt/4) | dc (Δt) - dc (Δt/4)

vertically integrated tracer mass difference (1h forecast)

Tracer advection

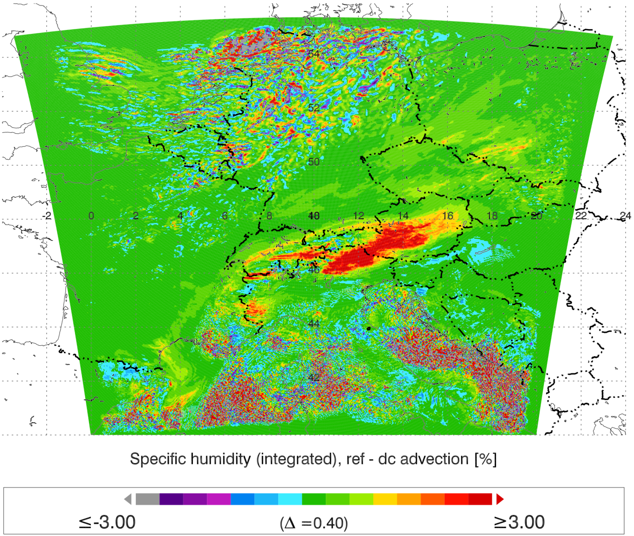


ref (Δt)- dc (Δt) | ref (Δt) - ref (Δt/2) | ref (Δt)- ref (Δt/4) | dc (Δt) - dc (Δt/4)

vertically integrated tracer mass difference (12h forecast)

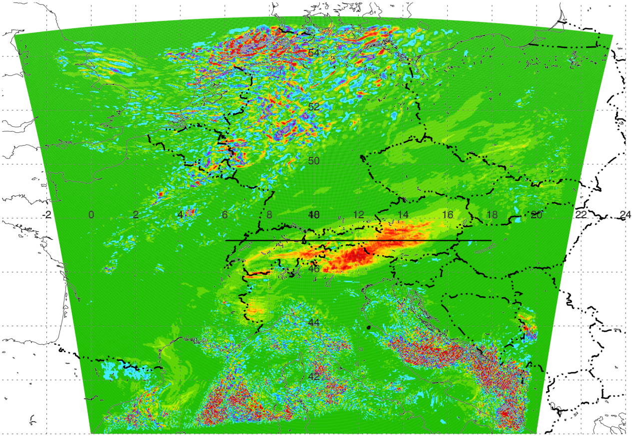
Results

Specific humidity difference (12h forecast)

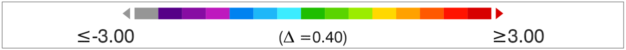


Results

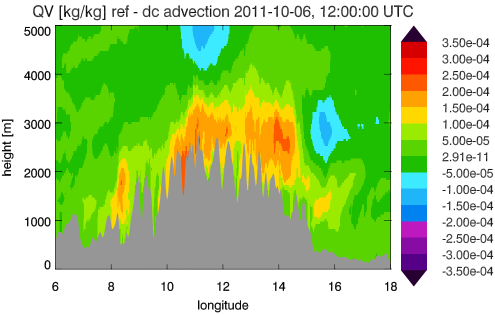
Specific humidity difference (12h forecast)

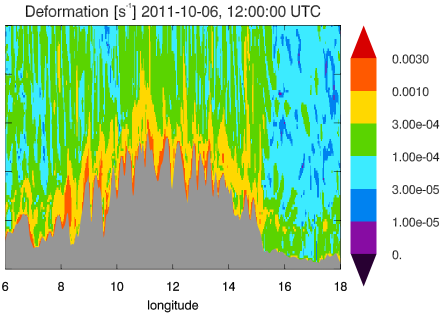


Specific humidity (integrated), ref - ref (Δt/4) advection [%]



Vertical cross section

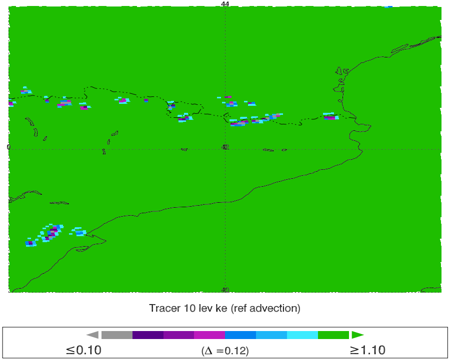




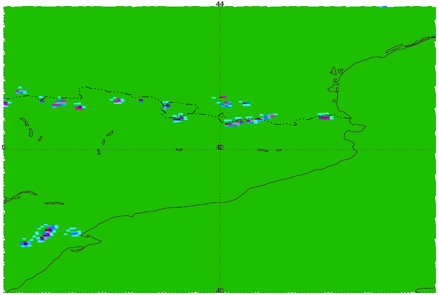
$$Def = \sqrt{\delta_{st}^2 + \delta_{sh}^2} \quad \text{with} \quad \delta_{st} = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}, \quad \delta_{sh} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

Results

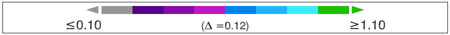
Mass implosions (05.01.12 00UTC +5min)

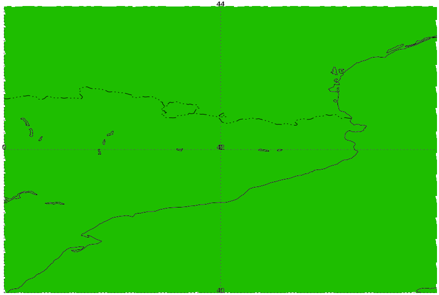


Mass implosions (05.01.12 00UTC +5min)

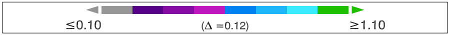


Tracer 10 lev ke (ref advection)





Tracer 10 lev ke (dc advection)



Summary

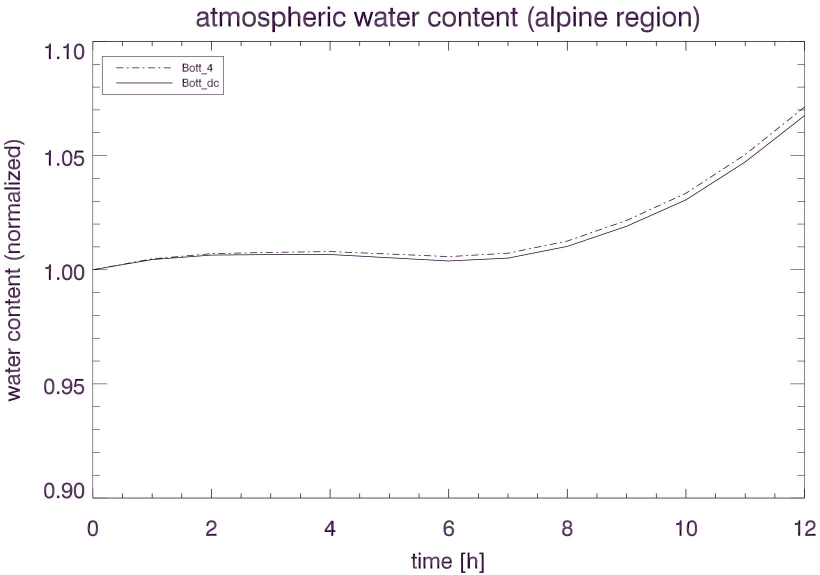
- Typical shortcomings of 1d flux-form advection schemes
  - results depend on time splitting approach
  - mass error due to diagnostic density
- Improvements with deformational correction
  - better representation of physical flow fields
  - hardly any dependency on time splitting approach
    $\rightarrow$ high accuracy without timestep reduction
  - improvement of mass balance
- Remaining problems
  - long term tests required to check improvement of model stability
  - time splitting errors are not completely corrected


Thank you for your attention!

Literature

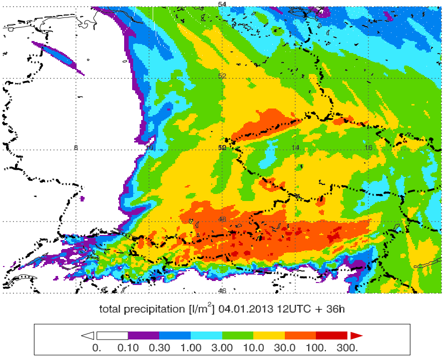
- Bott, A., Improving the time-splitting errors of one-dimensional advection schemes in multidimensional applications. Atmospheric Research, 97, pp619-631 (Sep. 2010)
- Schneider, W. and Bott, A ., On the time splitting errors of one-dimensional advection schemes in numerical weather prediction models (to be submitted)

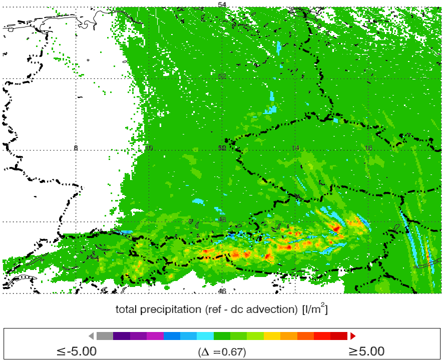
Results


atmospheric water content (alpine region)
Bott_4
Bott_dc
water content (normalized)
1.10
1.05
1.00
0.95
0.90
0
2
4
6
8
10
12
time [h]

Results

Effects on precipitation (orographic rain 05.01.2013)





Advection with deformational correction

upstream advection, Courant number 0.5 (x-y advection)

density

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

scalar [**/kg]

1	1	1	1
1	1	1	1
1	1	2	1
1	1	1	1

Advection with deformational correction

upstream advection, Courant number 0.5 (x-y advection)

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1	1	1	1
1	1	1	1

scalar [**/m³]

1	1	1	1
1	1	1	1
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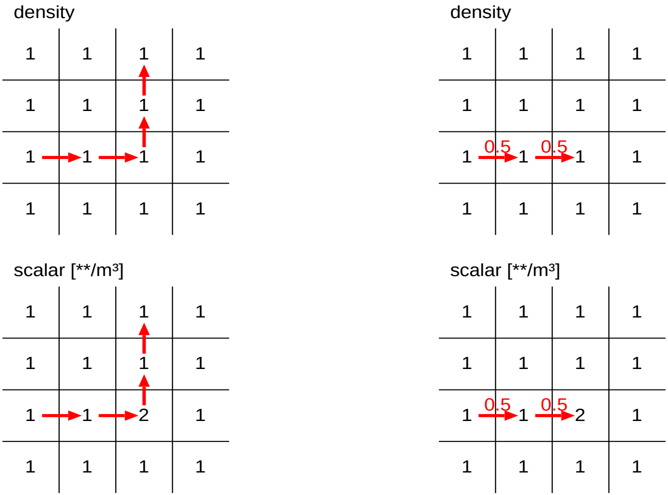
upstream advection, Courant number 0.5 (x-y advection)



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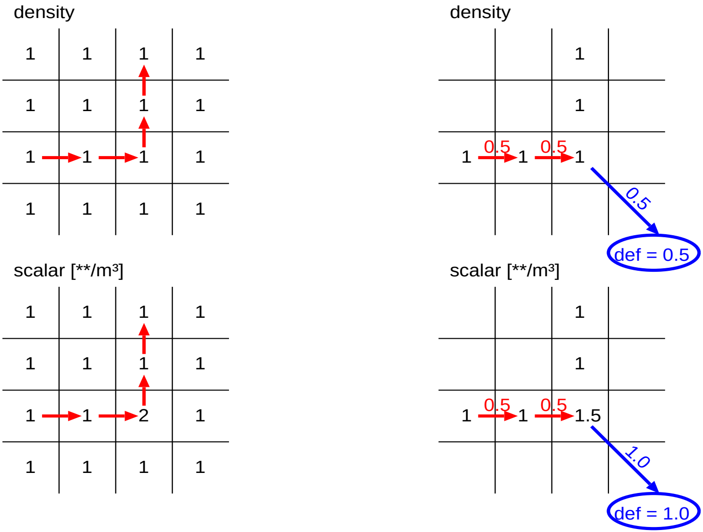

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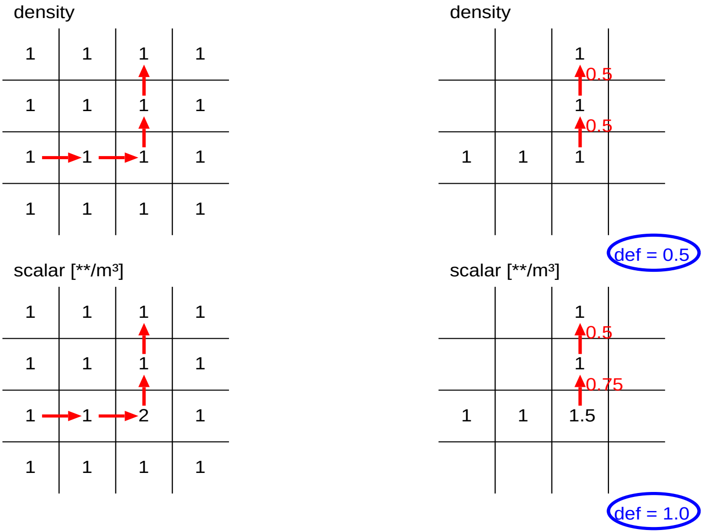
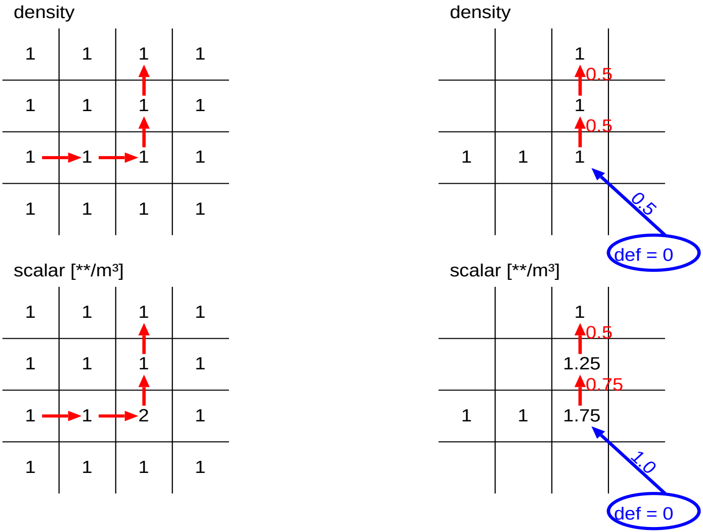


Advection with deformational correction

upstream advection, Courant number 0.5 (x-y advection)



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upstream advection, Courant number 0.5 (x-y advection)

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1	1	1	1
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1	1	1	1

density

		1	
		1	
1	1	1	

scalar [**/m³]

1	1	1	1
1	1	1	1
1	1	2	1
1	1	1	1

scalar [**/kg]

		1	
		1.25	
1	1	1.75	

Idealized flow field tests