# Verification of probabilistic products:
# New tools for the assessment of quantile forecasts

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**Acknowledgment:**
The German Federal Ministry for Economic Affairs and Energy: **EWeLiNE project** (Erstellung innovativer Wetter- und Leistungsprognosemodelle für die Netzintegration wetterabhängiger Energieträger)

# Outline:

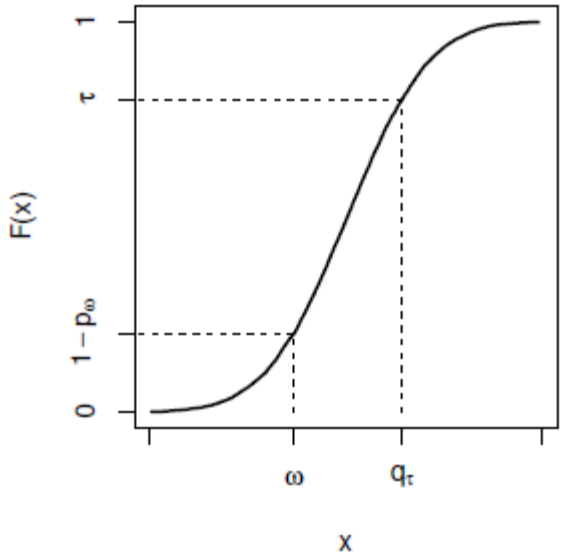
- Quantile forecast
- Cost-loss model
- User-based discrimination
- Quantile value plot

# Quantile Forecast

- **Ensemble forecasting and probabilistic products**

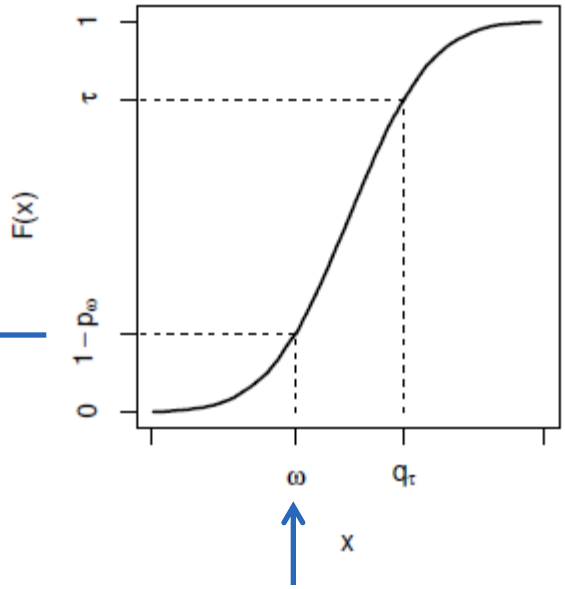
  Ensemble forecasts: realizations from a probability distribution

  F(x): cumulative probability distribution



- **Ensemble forecasting and probabilistic products**

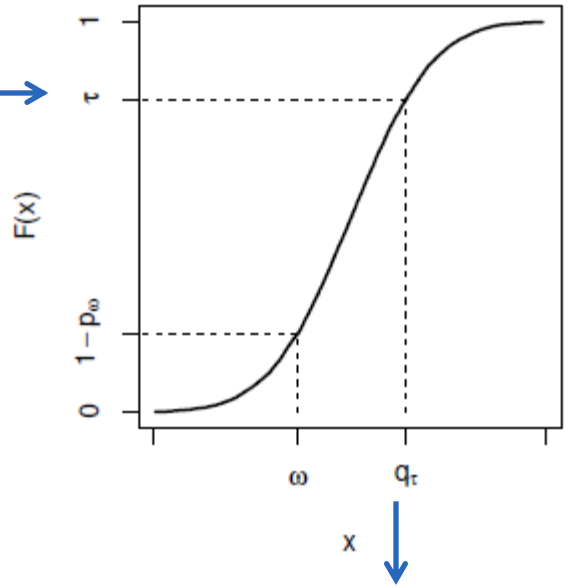
**Probability forecast** ←




**Event threshold**

- **Ensemble forecasting and probabilistic products**

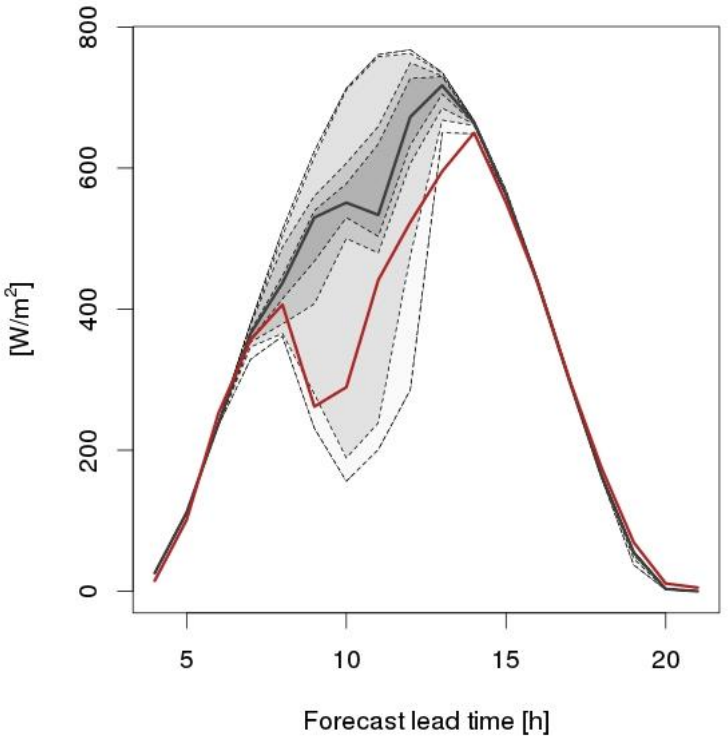
**Probability level** →




**Quantile forecast**

- **Quantile forecast:** a real example

**COSMO-DE-EPS**
**global radiation**
quantile forecasts and
observation (red) at “Arkona”,
July 2 2013



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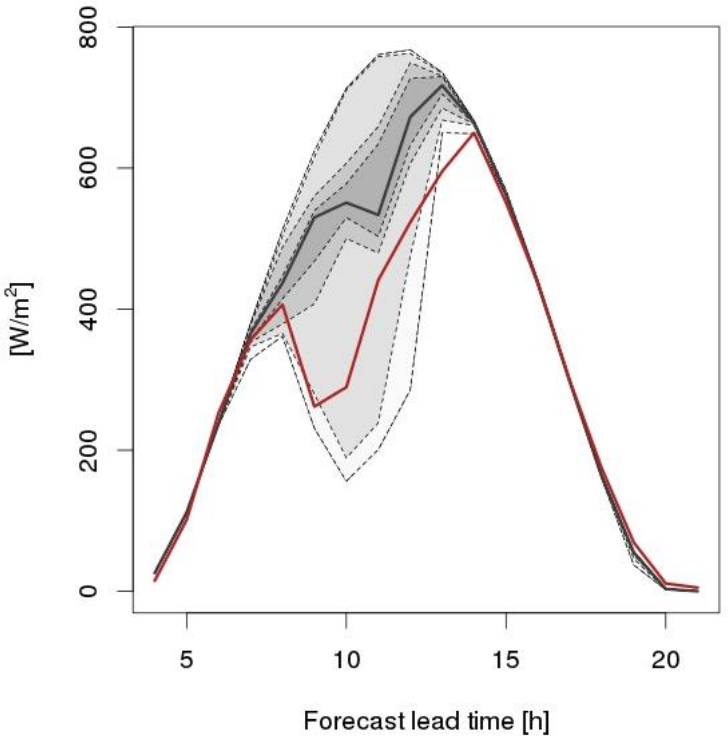


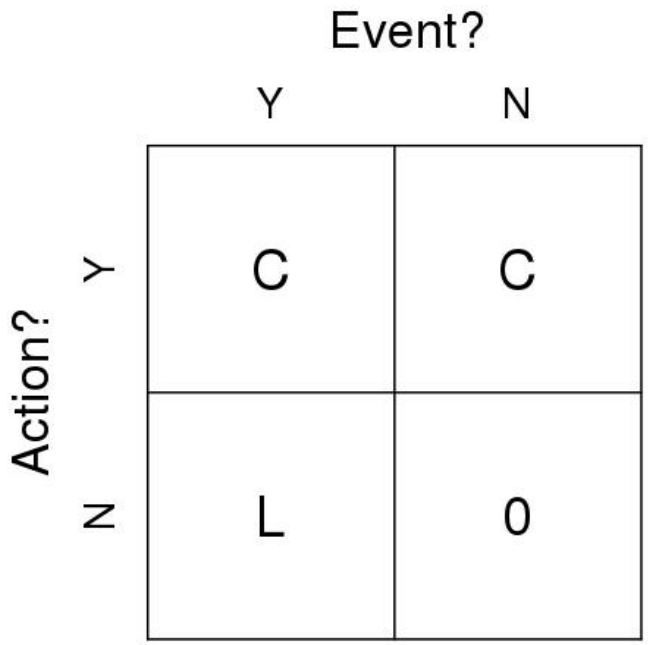

- **Key products for energy applications**

  Examples:
  - market participants who want to optimize their **bids**
  - system operators that have to optimize their **reserves**.

[Pinson, 2013]

# Cost-loss model

- **A decision-making framework: the cost-loss model**

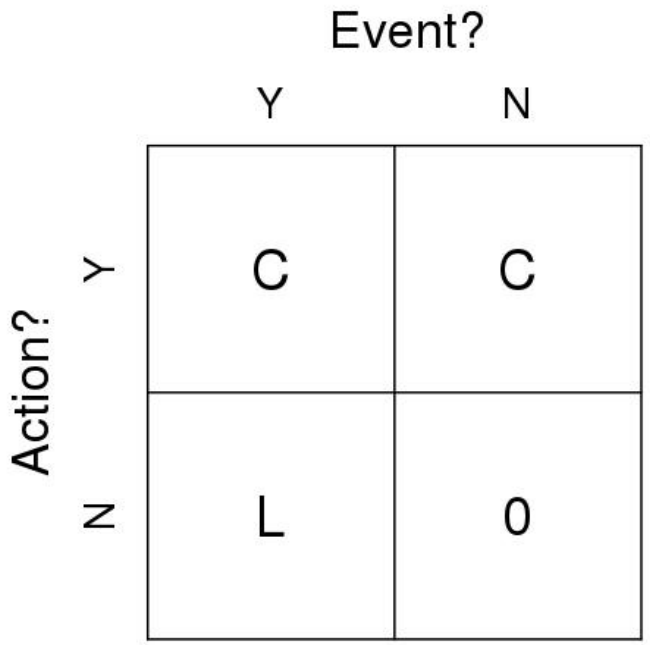


- **A decision-making framework: the cost-loss model**

Observation

**Event** $E : \Omega \geq \omega$

Event threshold



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**Action** $A : \Lambda \geq \lambda$

Action threshold

| Action? \ Event? | Y | N |
|---|---|---|
| Y | C | C |
| N | L | 0 |

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**Cost-loss ratio**

$$\alpha = \frac{C}{L}.$$

- **General framework: dichotomous cost-loss model**

Event

$$E : \Omega \geq \omega$$

User

$$\alpha = \frac{C}{L}.$$

**Optimal decision?**

$$A : \Lambda > \lambda_\alpha$$

- **General framework: dichotomous cost-loss model**

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[Richardson, 2011]

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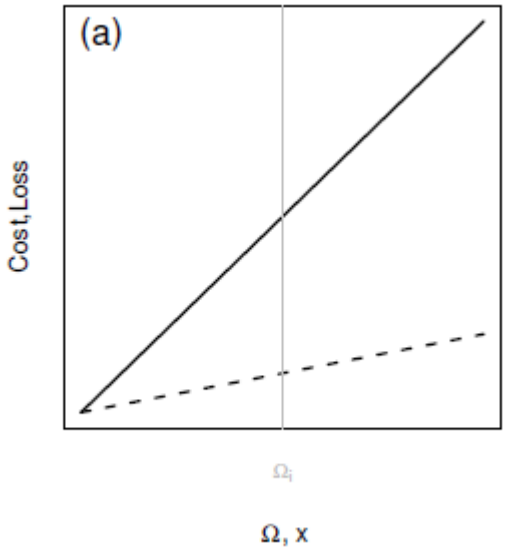
- **Continuous cost-loss model**

**C**: unitary cost, **L**:unitary loss



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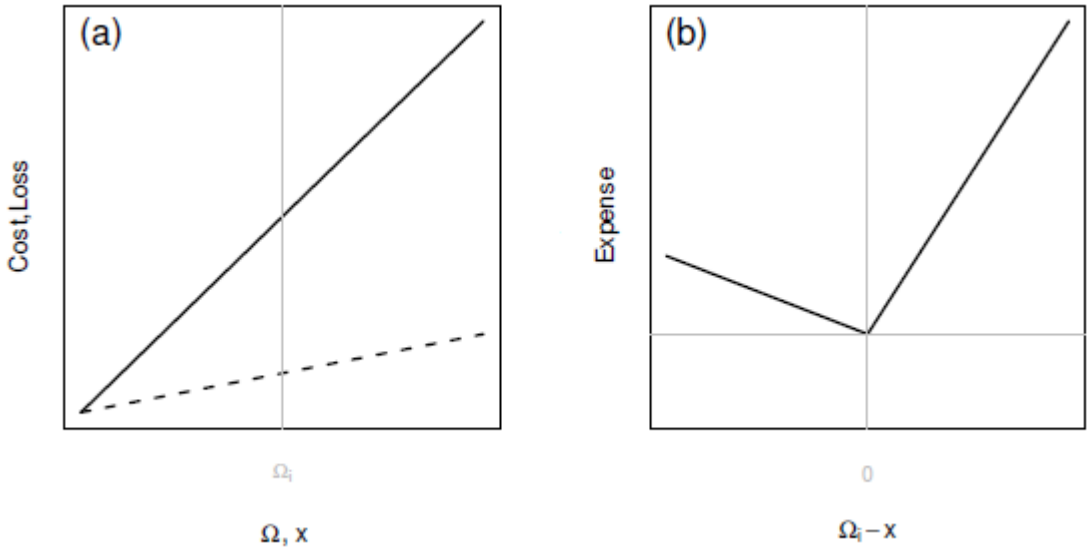
$$\text{Expense} = \begin{cases} C \cdot x & \text{if } \Omega_i \leq x \\ C \cdot x + L \cdot (\Omega_i - x) & \text{if } \Omega_i > x \end{cases}$$



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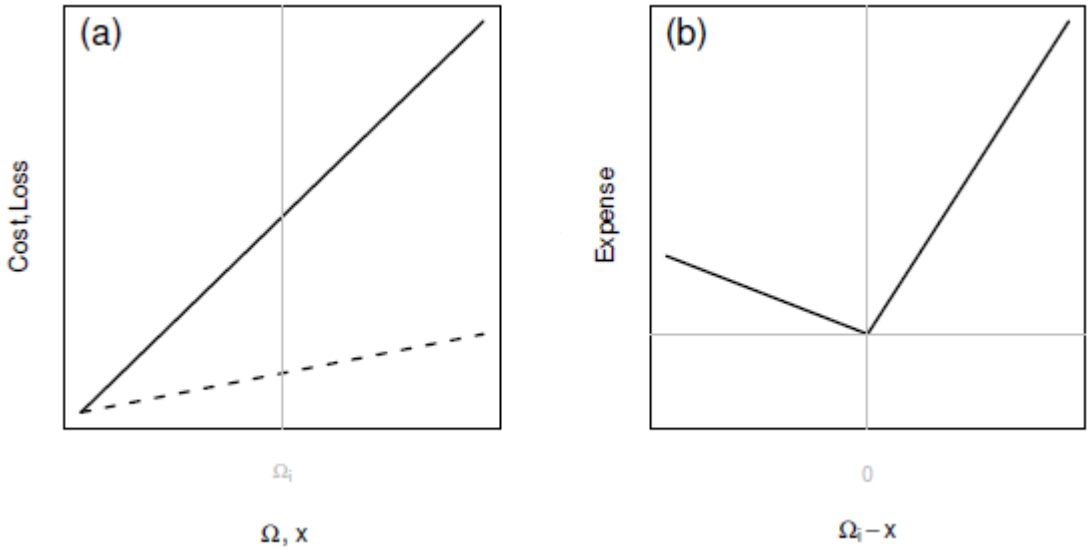


**Optimal forecast for users with asymmetric loss functions:**

quantile forecast at probability level $\tau = 1 - \alpha$

[Gneiting, 2011]

# User-based discrimination

- **User-based discrimination: the relative user characteristic (RUC) curve**

Focusing on a user:

$$H_\omega = Pr(\Lambda \geq \lambda_\alpha \mid \Omega \geq \omega)$$

$$F_\omega = Pr(\Lambda \geq \lambda_\alpha \mid \Omega < \omega)$$

No dicrimination:

$$H_\omega = F_\omega \quad \text{for all } \omega \in \Omega$$

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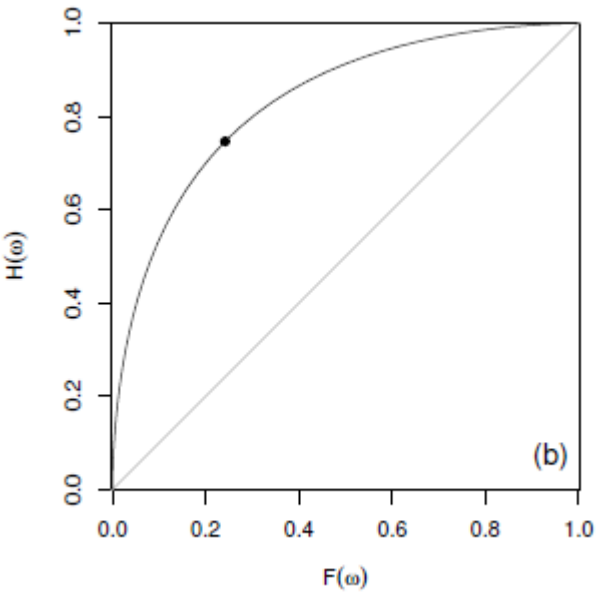

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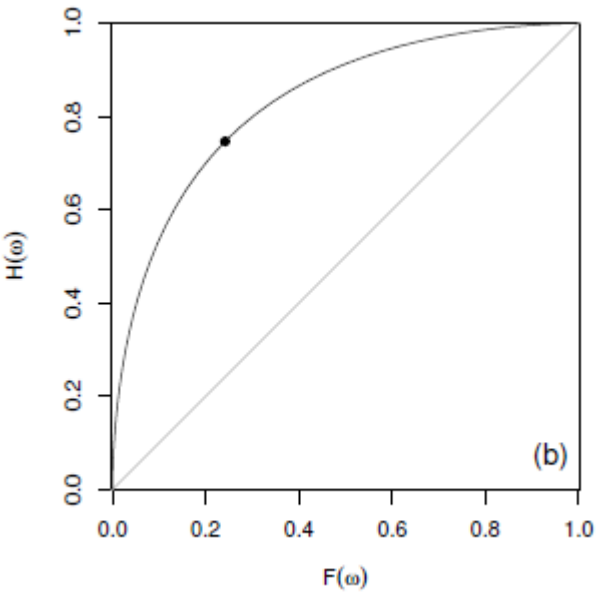


RUC is **user specific** but **event aspecific**
**RUC area** measures discrimination ability of quantile forecasts

# Quantile value plot

- **Value score** (based on the cost-loss model) **[Wilks, 2001]**

$$V = \frac{\bar{E}_{\text{climate}} - \bar{E}_{\text{forecast}}}{\bar{E}_{\text{climate}} - \bar{E}_{\text{perfect}}}$$

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Mean expense based on **climatological information**

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Mean expense based on **a forecast**

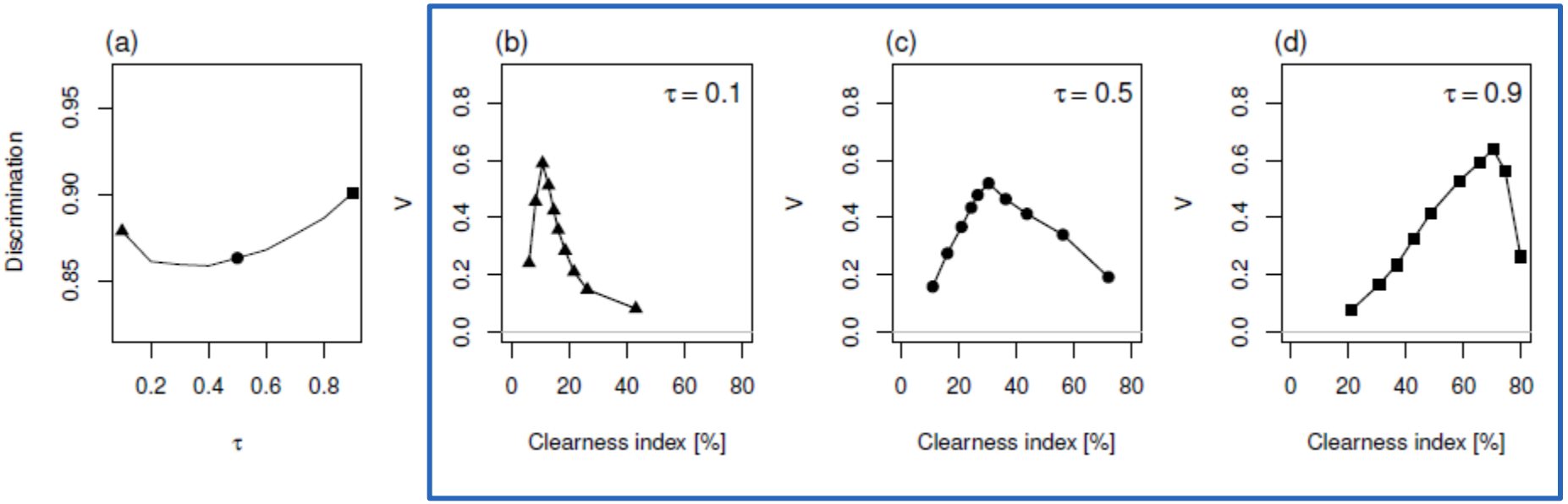
Mean expense based **on a perfect forecast**

$$V = \begin{cases} (1-F) - \left(\frac{\pi}{1-\pi}\right)\left(\frac{1-\alpha}{\alpha}\right)(1-H) & \text{if} \quad \alpha < \pi \\ H - \left(\frac{1-\pi}{\pi}\right)\left(\frac{\alpha}{1-\alpha}\right)F & \text{if} \quad \alpha > \pi. \end{cases}$$

**V** defined for an **event** (base rate) and a **user** (cost-loss ratio) function of the **forecast performance** in terms of H and F

$$H = Pr(\Lambda \geq \lambda \mid \Omega \geq \omega) \qquad F = Pr(\Lambda \geq \lambda \mid \Omega < \omega)$$

- **Quantile value plot**



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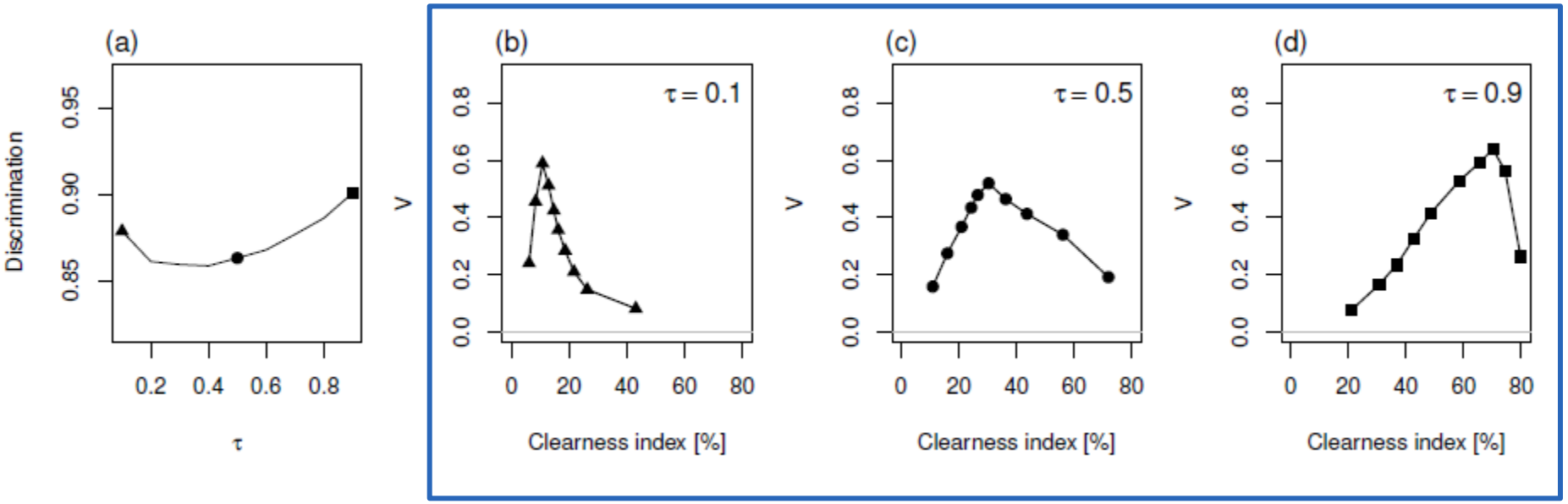


**Overall value** when the **distribution of event of interest is uniform** corresponds to the **quantile skill score** (with climatology as reference)

# CONCLUSION



| | Probability forecasts | Quantile forecasts |
|---|---|---|
| **Quality** | Bier score (BS)<br>**Brier,1950** | Quantile score (QS)<br>**Koenker and Machado, 1999** |
| **Reliability** | BS decomposition | QS decomposition |
| **Resolution** | BS decomposition<br>**Murphy,1973** | QS decomposition<br>**Bentzien and Friederichs, 2014** |
| **Discrimination** | ROC curve and area<br>**Mason, 1982** | |
| **Value** | Value score<br>**Wilks, 2001** | |

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**Wilks DS. 2001**. A skill score based on economic value for probability forecasts. *Met. App.*

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