Modelling Scalar Skewness in Cloudy Boundary Layers

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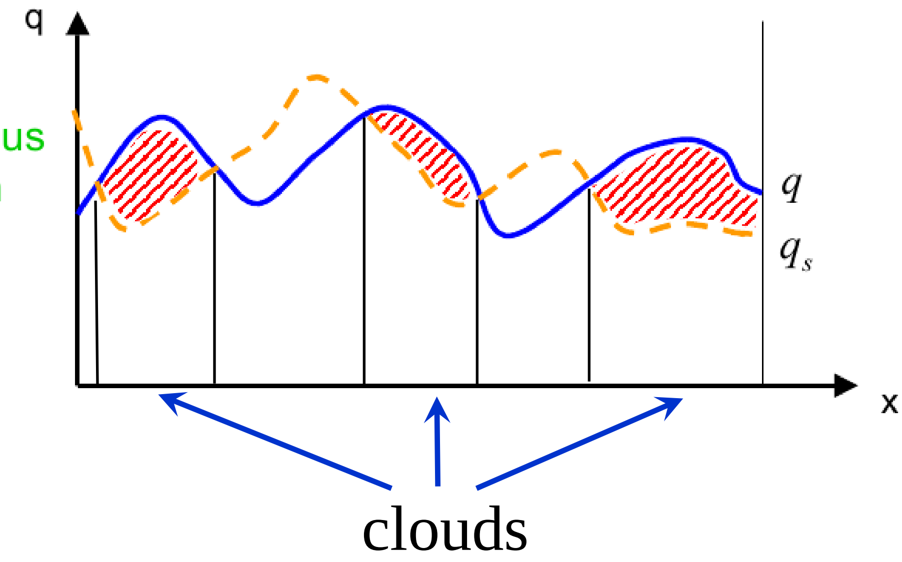
Outline

- Motivation (statistical cloud schemes, the need to account for skewness)
- Cloud scheme based on 3-parameter double-Gaussian PDF
- Transport equation for scalar skewness: derivation, closure assumptions, and coupling with the TKE-Scalar Variance (TKESV) mixing scheme
- Results from single-column tests
- Conclusions and outlook

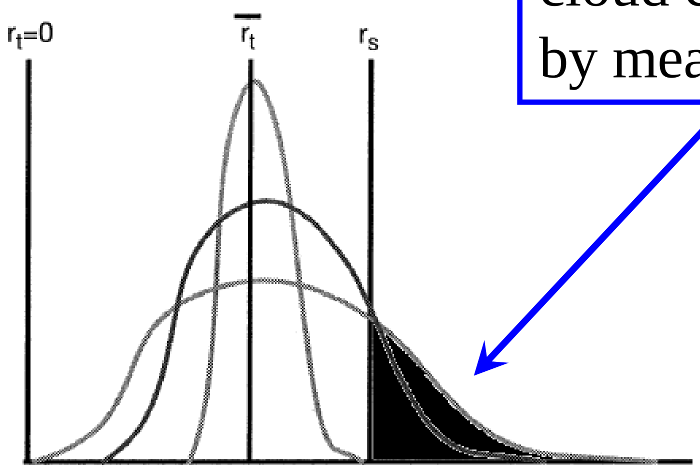
Recall…

Heterogeneous distribution of T and q



SGS fluctuations of q_t and q_s (due to SGS fluctuations of T) result in fractional cloud cover

after Tompkins (2002)



cloud cover C and cloud condensate $<q_l>$ are obtained by means of integration over a saturated part of PDF

If a PDF is assumed, e.g. $P(\theta_l, q_t)$, the problem essentially reduces to the prediction of PDF parameters (moments)

Statistical Cloud Schemes

Often formulated in terms of *linearized saturation deficit* (*s* variable of Mellor, 1977)

$$s = \frac{1}{Q}\left[\overline{q}_t + q_t' - q_s(\overline{T}_l) - \Pi \frac{\partial q_s}{\partial T}\bigg|_{\overline{T}_l} \theta_l' \right], \quad Q = 1 + \frac{L_v}{c_p}\frac{\partial q_s}{\partial T}\bigg|_{\overline{T}_l}, \quad \Pi = \frac{\overline{T}}{\overline{\theta}}$$

First-order moment of *s* is provided by the grid-scale equations

$$s = \frac{1}{Q}\left[\overline{q}_t - q_s(\overline{T}_l)\right]$$

Second-order moment of *s* should be provided by a turbulence scheme (e.g. TKE or TKESV)

$$\overline{s'^2} = \sigma_s^{\ 2} = \frac{1}{Q^2}\left[\overline{q_t'^2} - 2\Pi \frac{\partial q_s}{\partial T}\bigg|_{\overline{T}_l} \overline{q_t'\theta_l'} + \left(\Pi \frac{\partial q_s}{\partial T}\bigg|_{\overline{T}_l} \right)^2 \overline{\theta_l'^2} \right]$$

Higher-order moments?

Motivation

Two-parameter PDF in terms of mean saturation deficit normalized by its variance

$$Q_1 = \frac{\overline{q_t} - q_s(\overline{T_l})}{\sigma_s}$$

With the same mean and variance, cloud cover and the amount of cloud condensate vary as function of skewness.

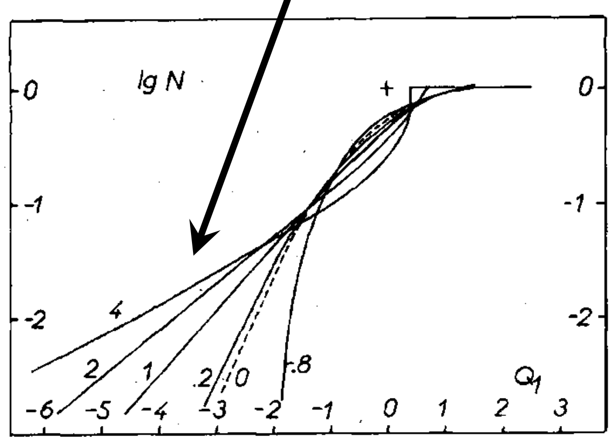


FIG. 1. Decimal logarithm of the predicted cloud cover. Symbols stand for the different values of the skewness factor. Dashed line: Gaussian scheme.

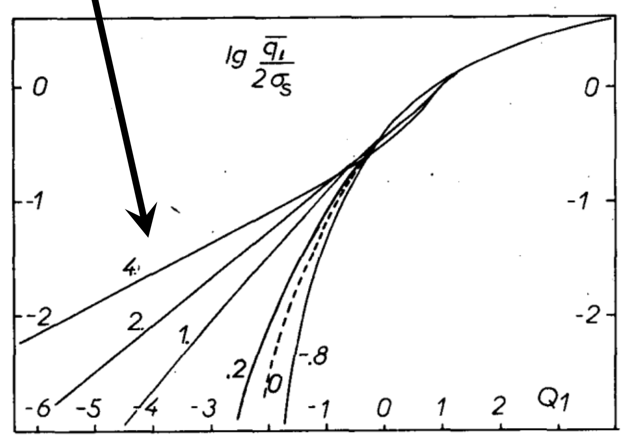


FIG. 2. Decimal logarithm of the normalized mean liquid water content. Legend as in Fig. 1.

(Bougeault, 1982)

In cumulus regime, the PDF asymmetry should be accounted for.

Motivation (cont'd)

Assumed PDFs used in cloud schemes (e.g. Larson et al., 2001)

One delta function (no variability), uniform (unfavorable shape) – poor

Gaussian, triangular – insufficient (symmetric)

Two delta – insufficient (ignore small-scale fluctuations)

Gamma, log-normal – insufficient (allow only positive skewness)

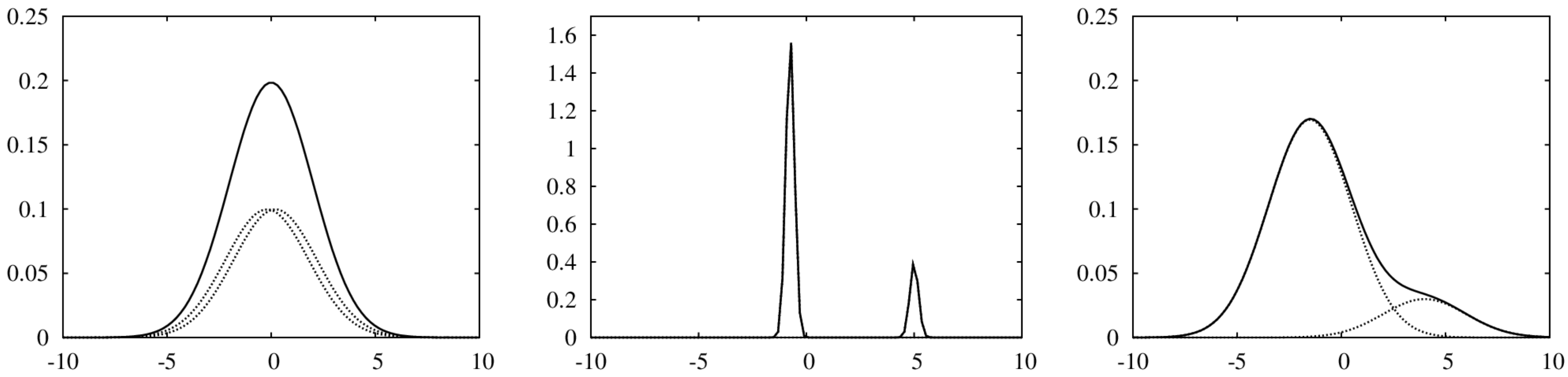
Beta – good, however unimodal

5-parameter double-Gaussian – remarkably good (very flexible, etc.), but complex and too computationally expensive

3-parameter double-Gaussian – good (flexible enough, etc.), likely an optimal choice

Double-Gaussian PDF

$$P(s) = \frac{a}{\sqrt{2\pi}\sigma_1} \exp\left[-\frac{1}{2}\left(\frac{s-s_1}{\sigma_1}\right)^2\right] + \frac{(1-a)}{\sqrt{2\pi}\sigma_2} \exp\left[-\frac{1}{2}\left(\frac{s-s_2}{\sigma_2}\right)^2\right]$$



Five parameters, viz., a, s_1, s_2 , σ_1 and σ_2 should be determined to specify a double-Gaussian PDF. To these end, five PDF moments should be predicted, e.g. the first five moments.

Remarkably good

but too complex and too computationally expensive!

3-Parameter Double-Gaussian PDF

Using LES data, Naumann et al. (2013) proposed

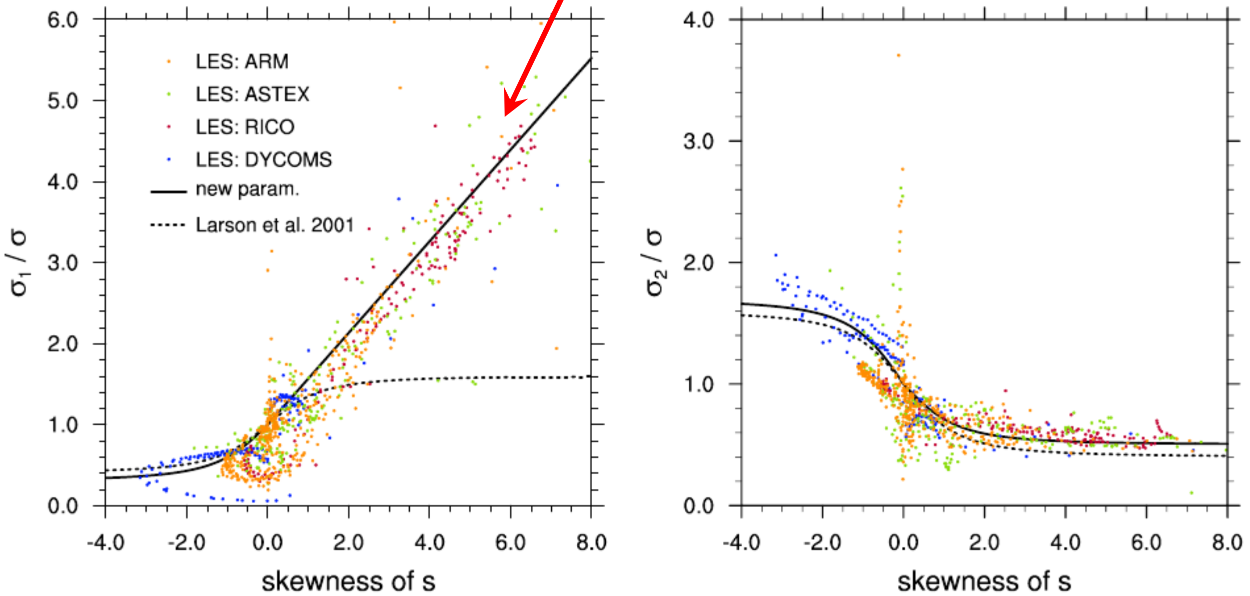
$S \leq 0$

$$\frac{\sigma_1}{\sigma} = 1 + \gamma_3 \frac{S}{\sqrt{\alpha + S^2}} \qquad \frac{\sigma_2}{\sigma} = 1 - \gamma_3 \frac{S}{\sqrt{\alpha + S^2}}$$

$S > 0$

$$\frac{\sigma_1}{\sigma} = 1 + \gamma_1 \frac{S}{\sqrt{\alpha}} \qquad \frac{\sigma_2}{\sigma} = 1 - \gamma_2 \frac{S}{\sqrt{\alpha + S^2}}$$

Cf. Larson et al. (2001)



Now σ_1 and σ_2 are functions of S and only 3 moments are needed to specify PDF.

Effect of Clouds on Buoyancy Production of TKE

Buoyancy term in the TKE equation

$$\frac{g}{\theta}\overline{w'\theta_v'} = \frac{g}{\theta}\left(A_* \cdot \overline{w'\theta_l'} + B_* \cdot \overline{w'q_t'} + D_* \cdot \overline{w'q_l'}\right)$$

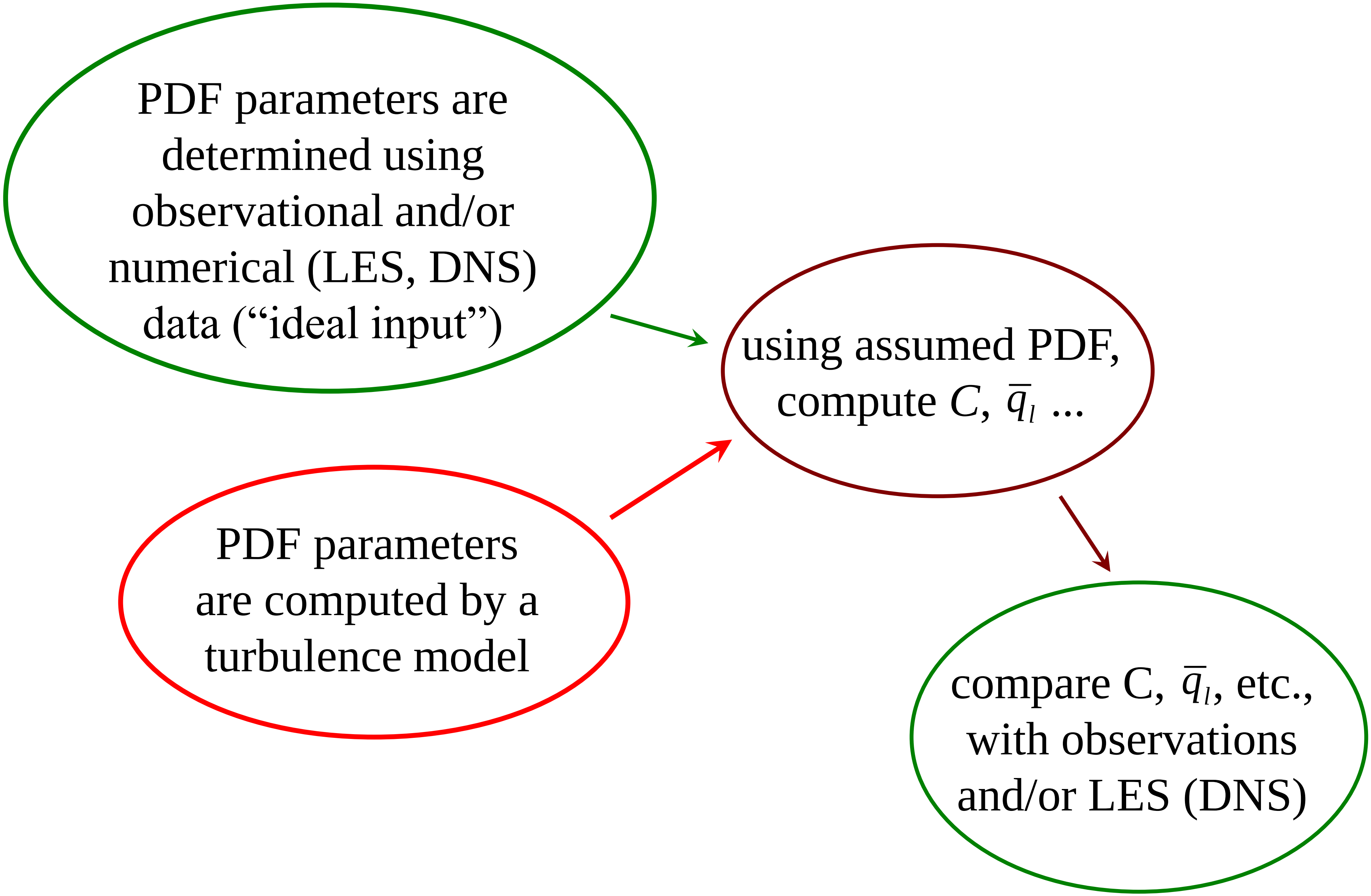
$\overline{w'q_l'}$ is unknown (no joint w-q_t-θ_l PDF), approximation is required. Naumann et al. (2013) proposed

$$\overline{w'q_l'} = FC\overline{w's'},$$

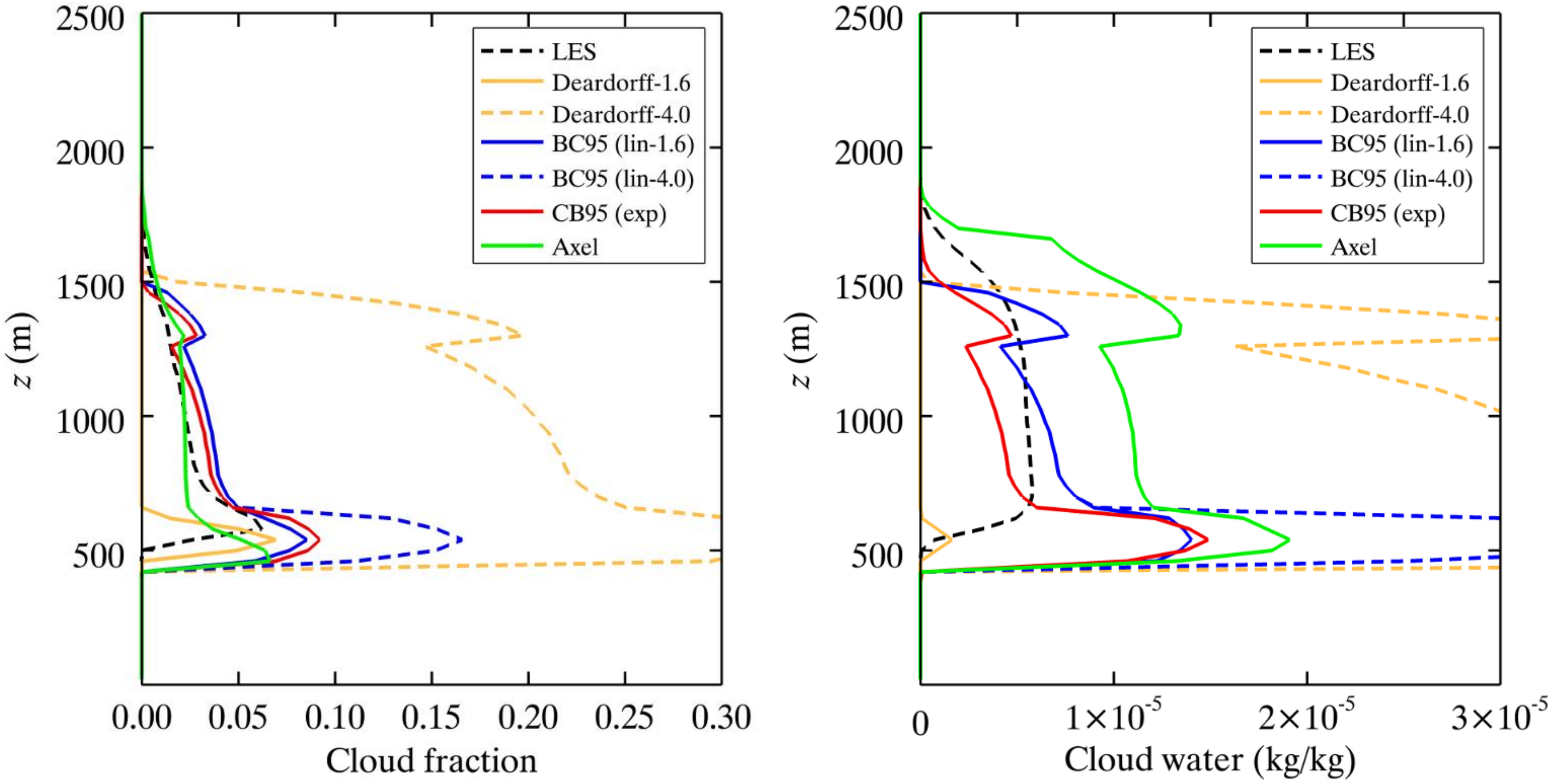
$$F(Q_1, S) = 1 + 1.5Q_1^2 \exp(0.25S) \quad \text{as} \quad Q_1 \le 0$$
$$F(Q_1, S) = 1 \quad \text{as} \quad Q_1 \ge 0,$$

where $Q_1 = \dfrac{\overline{q_t} - q_s(\overline{T_l})}{\sigma_s}$

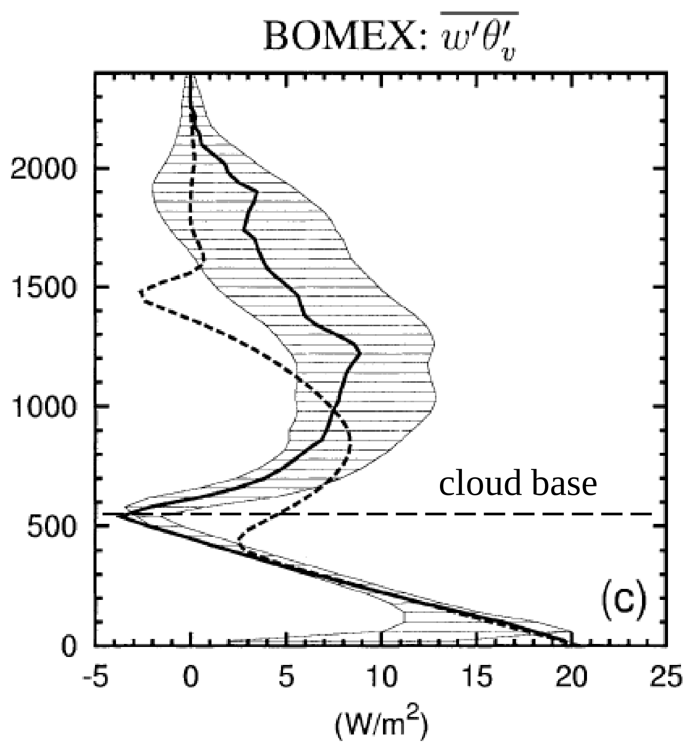
A Priori Testing of Cloud Schemes



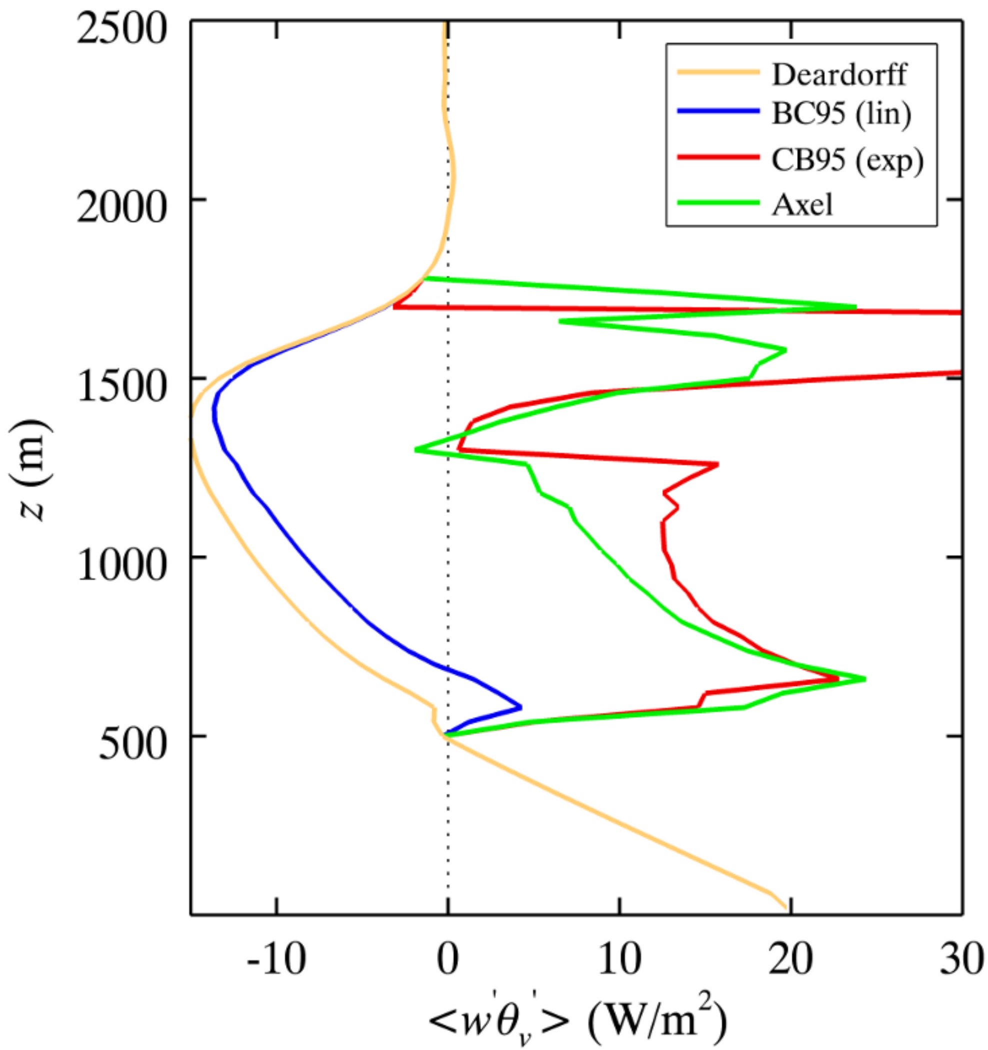
A Priori Testing: Cloud Fraction and Cloud Water



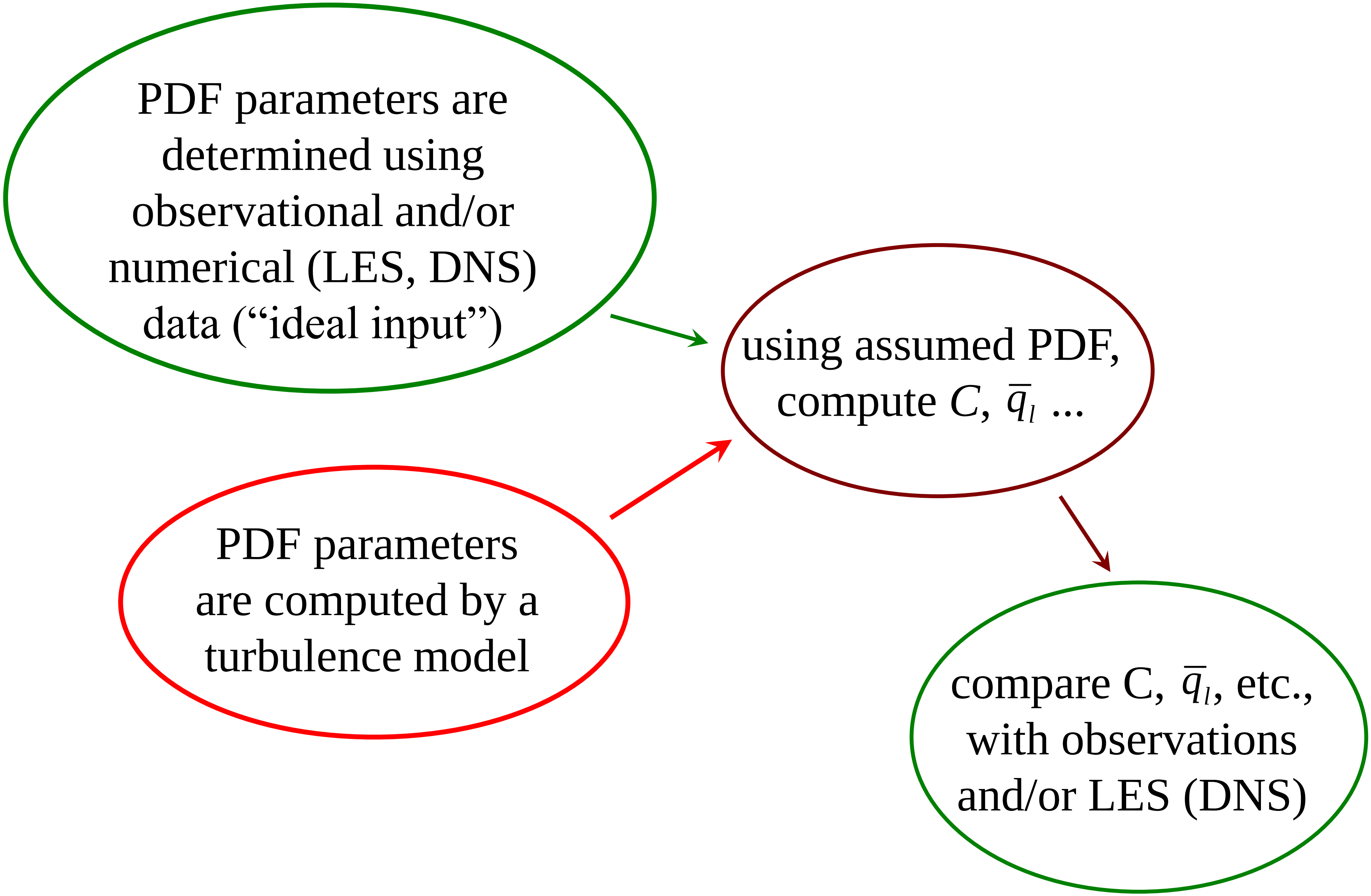
A Priori Testing: Expression for Buoyancy Flux (cloud fraction and cloud water are from LES)



LES data



Coupling with Turbulence Scheme



Transport Equation for Skewness ($\overline{s'^3}$)

The equation for the third-order moment of ***s***

$$\frac{1}{3}\left(\frac{\partial}{\partial t}+\bar{u}_i\frac{\partial}{\partial x_i}\right)\overline{s'^3} = -\underline{\overline{u_i's'^2}}\frac{\partial \bar{s}}{\partial x_i}+\overline{s'^2}\frac{\partial \overline{u_i's'}}{\partial x_i}-\frac{1}{3}\frac{\partial}{\partial x_i}\underline{\overline{u_i's'^3}}-\underline{\varepsilon_s}$$

Closure is required for dissipation rate $\boldsymbol{\varepsilon_s}$,

third-order moment $\overline{\boldsymbol{u_i's'^2}}$, and fourth-order moment $\overline{\boldsymbol{u_i's'^3}}$

Dissipation rate: $\varepsilon_s = \dfrac{\overline{s'^3}}{\tau}$, $\tau = \dfrac{l}{C_\varepsilon\sqrt{e}}$, *l* is the length scale

Equation for $\overline{s'^3}$: Closure Assumptions

Third-order moment $\overline{u_i's'^2}$ (Mironov et al. 1999, Gryanik and Hartmann 2002)

small-scale random fluctuations (≈ Gaussian)

$$\overline{u_i's'^2} = -K\frac{\partial \overline{s'^2}}{\partial x_i}$$

PBL-scale coherent structures (two-delta function = mass-flux)

$$\overline{u_i's'^2} = S\left(\overline{s'^2}\right)^{1/2}\overline{u_i's'}$$

interpolation

$$\overline{u_i's'^2} = -K\frac{\partial \overline{s'^2}}{\partial x_i} + S\left(\overline{s'^2}\right)^{1/2}\overline{u_i's'}$$

As the resolution is refined, the SGS motions are (expected to be) increasingly Gaussian.

Then, $S \rightarrow 0$ and the parameterization of the third-order transport term reduces to the down-gradient diffusion approximation.

Equation for $\overline{s'^3}$: Closure Assumptions (cont'd)

Fourth-order moment $\overline{{u_i}'s'^3}$ (Gryanik and Hartmann, 2002)

Gaussian formulation

$$\overline{{u_i}'s'^3} = 3\overline{s'^2}\,\overline{u_i's'}$$

two-delta function (=mass-flux)

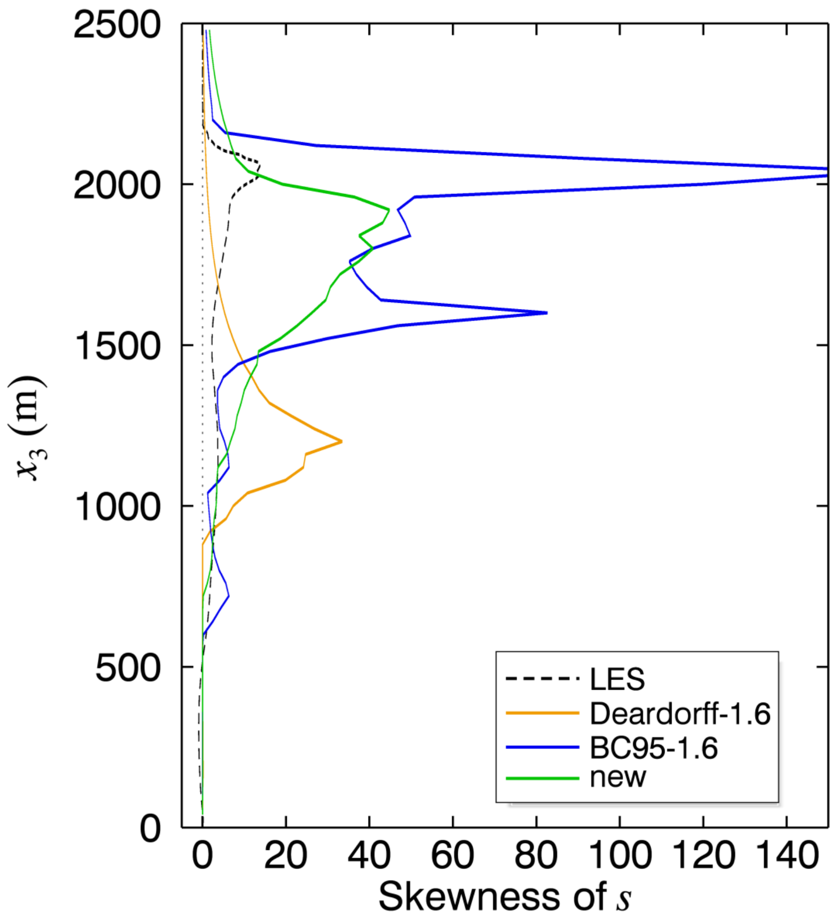
$$\overline{{u_i}'s'^3} = 3S^2\overline{s'^2}\,\overline{u_i's'}$$

interpolation

$$\overline{{u_i}'s'^3} = 3(1+S^2)\overline{s'^2}\,\overline{u_i's'}$$

No need for equations of higher order!

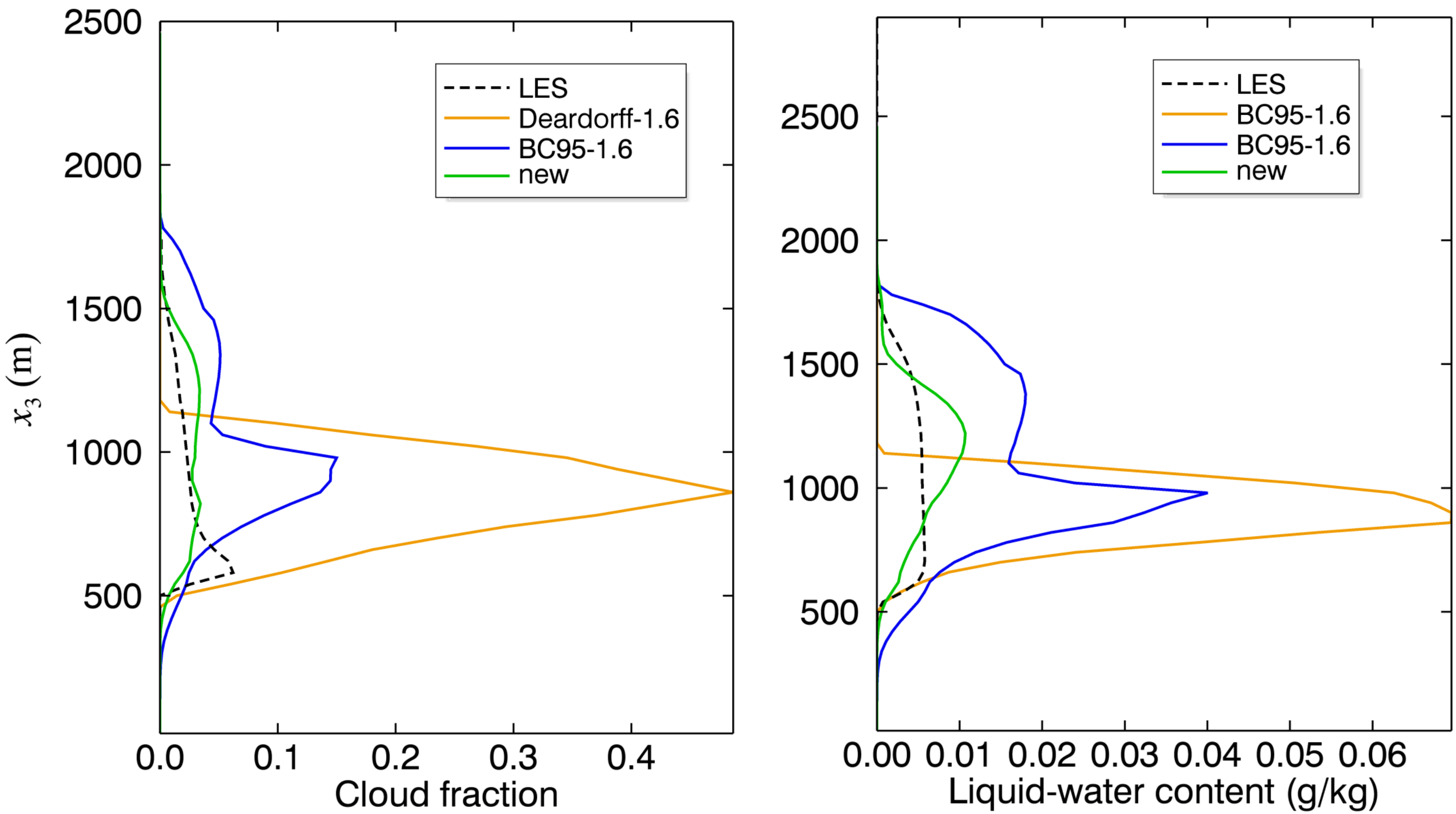
TKESV + NSM (new) Cloud Scheme: Skewness



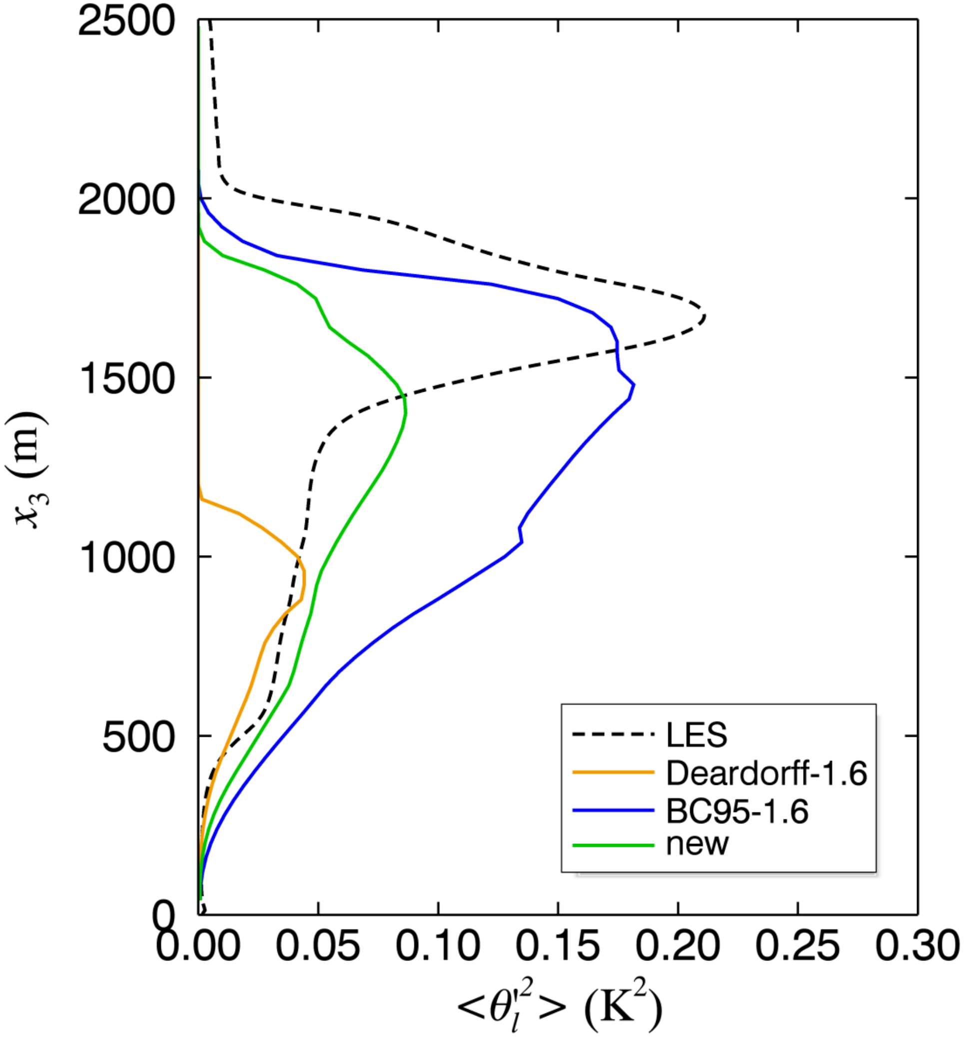
BOMEX shallow cumulus test case (http://www.knmi.nl/~siebesma/BLCWG/#case5) . Profiles are computed by means of averaging over last 3 hours of integration (hours 4 through 6). LES data are from Heinze (2013).

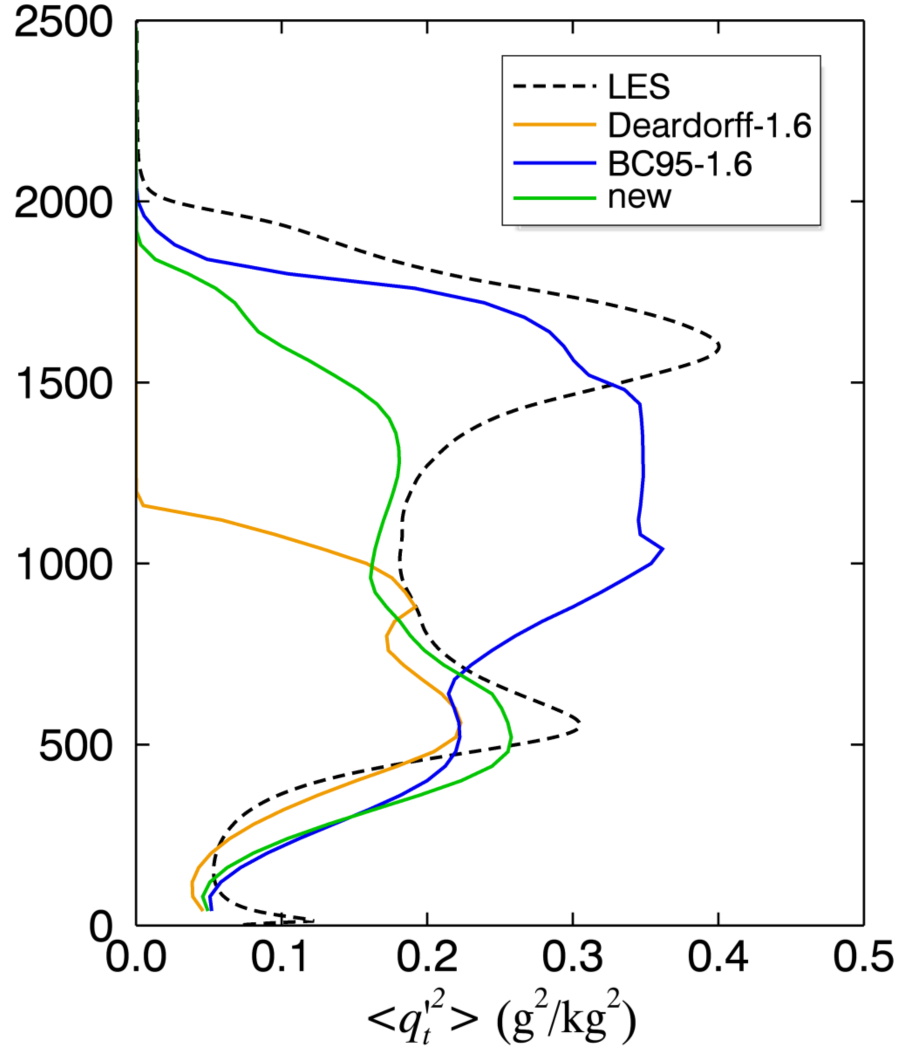
Second-order moments are computed with the TKE-Scalar Variance scheme (Machulskaya and Mironov, 2013).

TKESV + NSM (new) Cloud Scheme: Cloud Fraction and Cloud Water

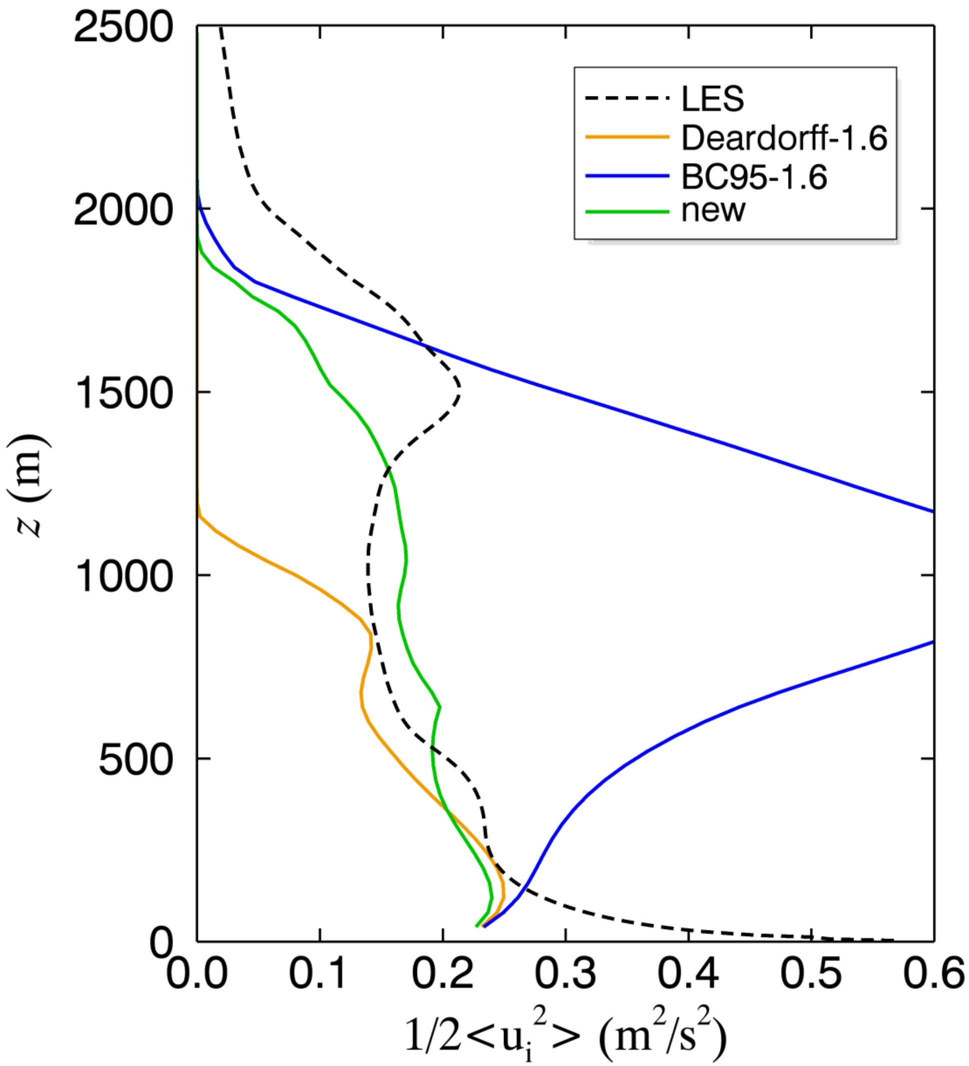


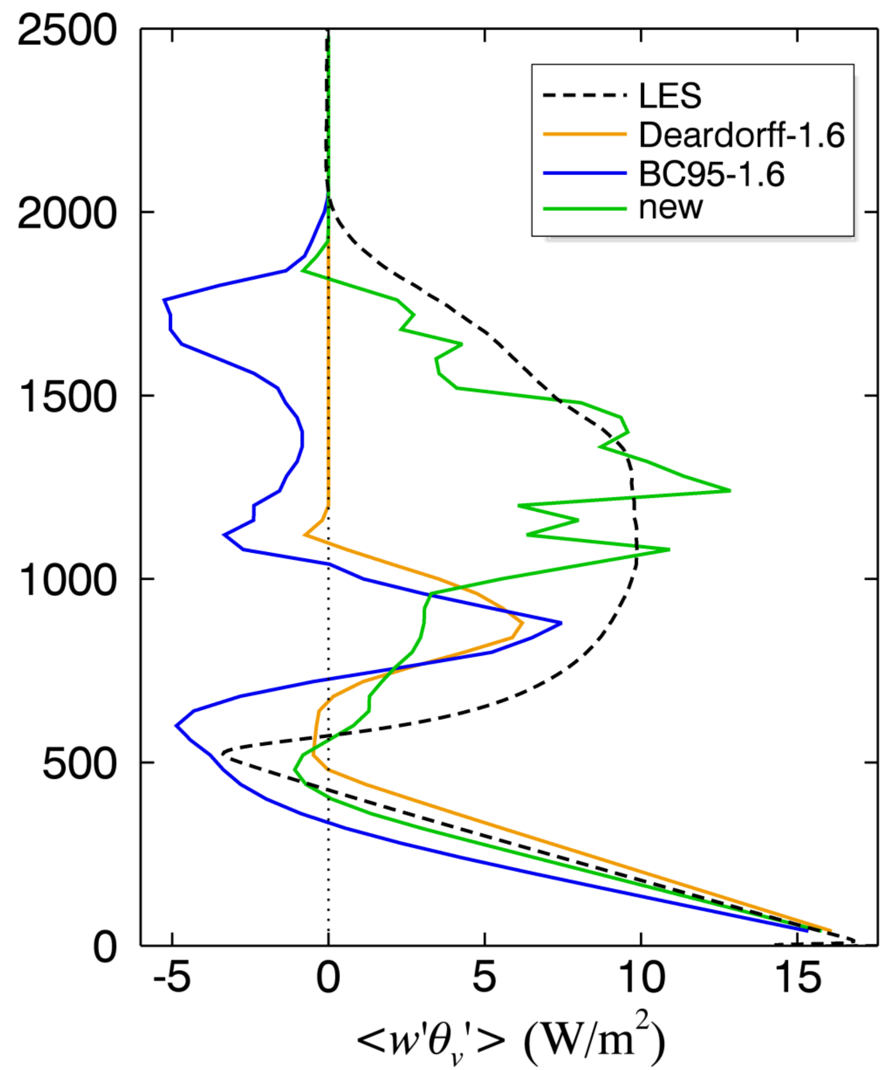
TKESV + NSM (new) Cloud Scheme: Temperature and Humidity Variance





TKESV + NSM (new) Cloud Scheme: TKE and Buoyancy Flux





Conclusions and Outlook

- Transport equation for the skewness of linearized saturation deficit (*s*) is developed
- The *s*-equation is coupled to the TKESV mixing scheme and to the statistical cloud scheme based on a 3-parameter double-Gaussian PDF
- First results from single-column tests look promising

- Further testing, delicate issues (stratus clouds, SBL, etc.)
- Implementation into COSMO and ICON
- Effect of microphysics on scalar variance and skewness

Thank you for your kind attention!

Acknowledgements: Rieke Heinze and Siegfried Raasch (LES data)

NB: joint PDF including *w*

To <u>consistently</u> determine vertical fluxes $\overline{w'\theta_l'}$ and $\overline{w'q_t'}$,

and the buoyancy flux

$$\frac{g}{\theta}\overline{w'\theta_v'} = \frac{g}{\theta}\left(A \cdot \overline{w'\theta_l'} + B \cdot \overline{w'q_t'} + D \cdot \overline{w'q_l'}\right)$$

a joint distribution $P(w', q_t', \theta_l')$ is needed (e.g. Golaz et al. 2002).

Then

$$\overline{w'\theta_l'} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w'\theta_l' P(w', \theta_l', q_t') dw' d\theta_l' dq_t'$$

Buoyancy Flux (PDF does not include *w*)

$\overline{w'\theta_v'}$ can be obtained without further assumptions for **clear-sky ("dry")** and **overcast ("wet")** grid boxes

$$\overline{w'\theta_v'}\Big|_{dry} = A_{dry}\cdot\overline{w'\theta_l'} + B_{dry}\cdot\overline{w'q_t'}$$

$$\overline{w'\theta_v'}\Big|_{wet} = A_{wet}\cdot\overline{w'\theta_l'} + B_{wet}\cdot\overline{w'q_t'}$$

for "wet",

$$q_l = q_t - q_s(T_l)$$

is used

Interpolation: $$\overline{w'\theta_v'} = (1-R)\cdot\overline{w'\theta_v'}\Big|_{dry} + R\cdot\overline{w'\theta_v'}\Big|_{wet}$$

R is close to the cloud fraction C for Gaussian PDF

$R \approx C$ does not hold in many situations, e.g. for cumulus clouds

(C is small but $\overline{w'\theta_v'}$ is dominated by $\overline{w'\theta_v'}\Big|_{wet}$)

"Non-Gaussian" correction is required to compute the buoyancy flux!

Formulation of Naumann et al. (2013)

In terms of liquid water flux

$$\overline{w'q_l'} = FC\overline{w's'}$$

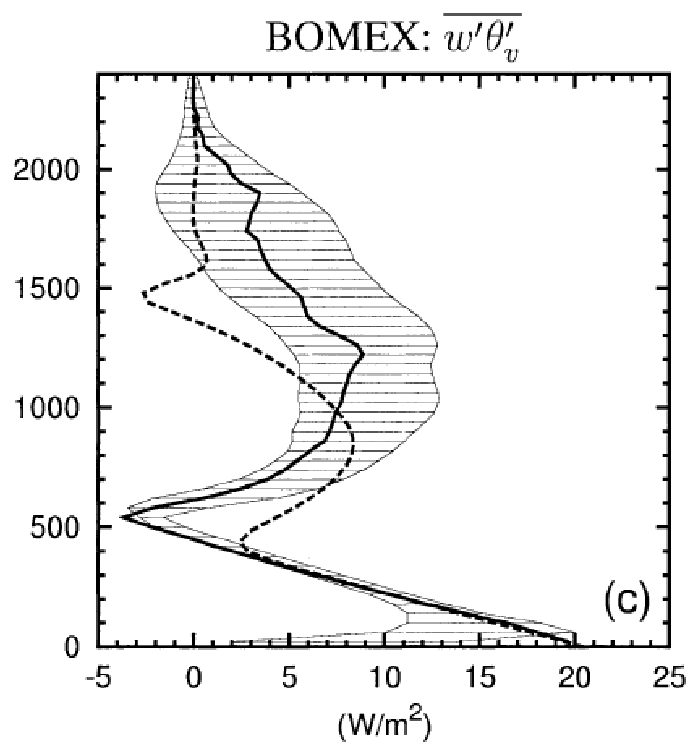
Approximation of F with due regard for skewness S

$$F\left(Q_1, S\right) = 1 + 1.5Q_1^2 \exp\left(0.25S\right) \quad \text{as} \quad Q_1 \le 0$$

$$F\left(Q_1, S\right) = 1 \quad \text{as} \quad Q_1 \ge 0,$$

where $Q_1 = \dfrac{\overline{q_t} - q_s(\overline{T_l})}{\sigma_s}$

A Priori Testing: Expression for Buoyancy Flux
(cloud fraction and cloud water are diagnosed from assumed PDF)



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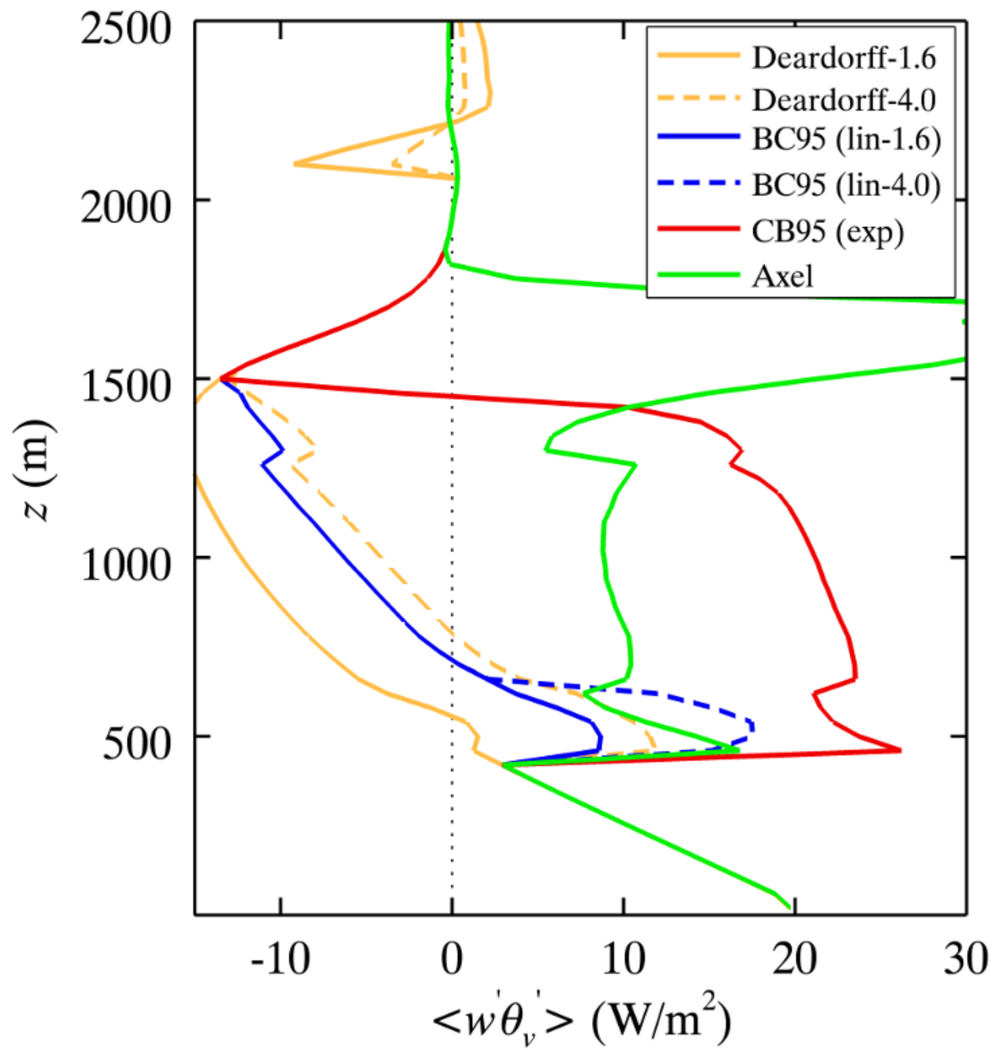

Third-Order Moment $\overline{s'^3}$

Recall: linearized saturation deficit is defined as

$$s = \frac{1}{Q}\left(\overline{q_t} + q_t' - q_s(\overline{T_l}) - \Pi\frac{\partial q_s}{\partial T}\theta_l{}'\right) \qquad Q = 1 + \frac{\mathcal{L}_v}{c_p}\frac{\partial q_s}{\partial T} \qquad \Pi = \frac{\overline{T}}{\overline{\theta}}$$

$$\Longrightarrow \qquad s' = \frac{1}{Q}\left(q_t' - \Pi\frac{\partial q_s}{\partial T}\theta_l{}'\right)$$

Four third-order moments are required to determine $\overline{s'^3}$ (i.e. four transport equations – too expensive). We carry just one transport equation, viz., the equation for $\overline{s'^3}$.

Note that the time rate-of-change and advection of Q and Π are neglected. This is not a principal assumption, it just makes the treatment of the third-order moment simpler.