

# Faster Models by Sparse Grids, Improved Physical Interfaces and Larger Time-steps

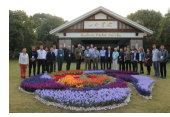
J. Steppeler<sup>1</sup>, J. Li(李锦熙)<sup>2</sup>, F. Fang<sup>3</sup>

<sup>1</sup> MoW GmbH: Spessartstr 10, Bad Orb, Germany

<sup>2</sup> Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing, China 中国科学院大气物理研究所, 中国北京  
<sup>3</sup> Applied Mathematics and Computing Group, Imperial College London, London, United Kingdom



Announcements of  
 MoW, Bad Orb, Germany 2019  
 &  
 AMCA, Shanghai, China, 2018



Participants of AMCA, China, 2018



Participants of MoW, France, 2015



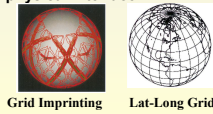
The Great Wall with J. Li



Dinner with Mayoress of Erqu, France

## Advantages of Spectral Element Method

- Uniform approximated order higher than two under the original icosahedral grid
- o3o3 saves computer time by sparseness and better physical interface
- Mass conservation and without grid imprinting
- Polynomials fit continuously at boundaries
- Easily brought to multiprocessor systems



## Advantages of Polygonal Spline Interpolation

- For computational efficiency: Inhomogeneity of older methods (???) costs efficiency and reduces the allowable time step
- For parallel computing: easily brought to multiprocessor systems and scales well
- For coupling with physics: easily applied to many numerics schemes with uniform-order: FDM, SEM FEM, o3o3

## onom method of L-Galerkin Method for Regular Reso

- L-Galerkin methods such as SEM sparse, o3o3 combine high approximation order with conservation and suitability for irregular resolution
- 1st o: the fields represented by  $n$ -th-degree polynomials
- 2nd o: the fluxes represented by such of degree  $m$
- Grid of o3o3 Method for Regular Resolution:

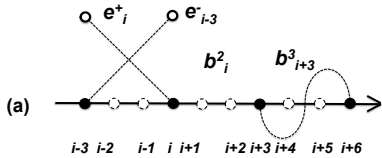


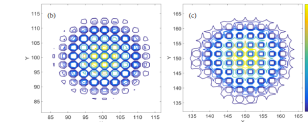
Figure1:  
 o3o3 computational grid: Regular points  
 SEM grid: Irregular Gauss-Lobatto points

## Advantages of onom method:

- Release time-step restriction in SEM
- Improve computational efficiency by sparseness
- Accelerate physics-dynamics coupling by spline physical interface

## Result 1: Time Step in Advection Experiments

- o3o3 advection with a homogeneous velocity field



- The plot gives a blowup of the fields belonging to (b) the initial time and (c) the 50th time step

Schemes	CFL condition
Standard o4	2.0
o3o3 standard	2.5
o3o3 spectral	2.2
3rd SEM	1.0

Table:  
 CFL condition with RK4 time-stepping

## Result 2: Sparseness of Grid by onom method

- Sparseness factor being the relation of the points of the full grid to the points of the sparse grid is an indication of potential computer saving

Numerics	o2o3 square	o2o3 hexagon	o3o3 square	o3o3 hexagon
2-D	4:3	12:6	9:5	27:2
3-D	8:4	24:8	27:7	81:8

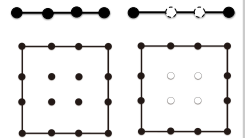


Figure: Computational grid in 2D.

(a) full grid and (b) sparse grid where unused points are indicated in white.

## Result 3: Spline Physical Interface

- Why: Traditional introduction of physical processes into numerical models: Call the parameterization at every grid point → expensive computing + sampling errors
- How: (An example: a condensation experiment)
- Condensation of Moisture:  $h(x) = \min [2.0, h(x)]$
- Calling the equation at the boundary points of intervals in o3o3 while applying cubic spline interpolation to the interior points to form sparseness of grids

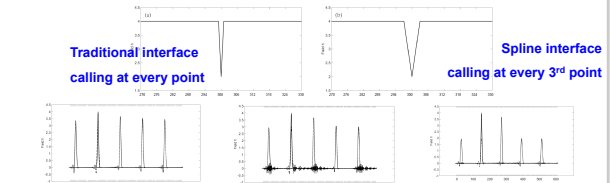


Figure: Advection (c) without condensation, (d) with traditional interface and (e) with spline interface

## onom Method for Irregular Resolution

- onom is also of uniform order with irregular grids and easily applies to icosahedral or cubed sphere grids without using grid smoothing

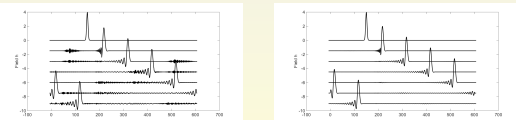
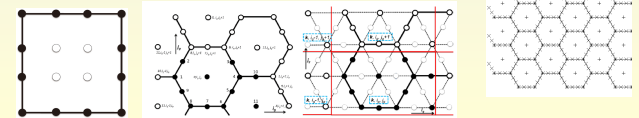


Figure: Advection in irregular resolution (left) control run by standard 4th order scheme, (right) o2o3 method

- 在两到三个月之内, o2o3 局地伽辽金方法能够为几乎任何数值模式提高4倍的计算速度。

## Computer Saving from Sparseness and Physical Interface

Sparseness Factor (for saving in 3-D)	Square	Interface	Time step
o3o3 standard	7:27	7:56	7:81
o3o3c1	4:27	4:56	4:81
o3o3c1_Hexagon	6:81	3:81	1.5:81



The secret of MoW&more is hard work day and night

日以继夜的不断努力是MoW获得成功的秘诀



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