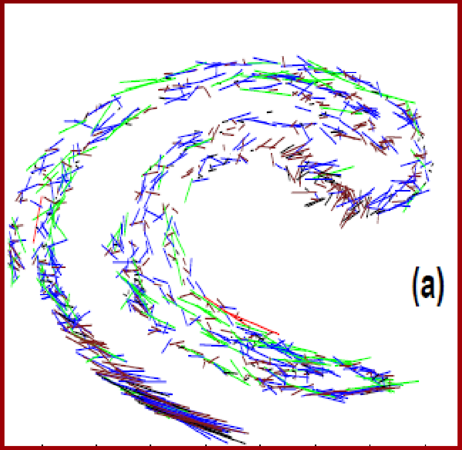
Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Pseudo Orbit Data Assimilation (PDA)



PDA cost function

Perfect & Imperfect Model Scenario

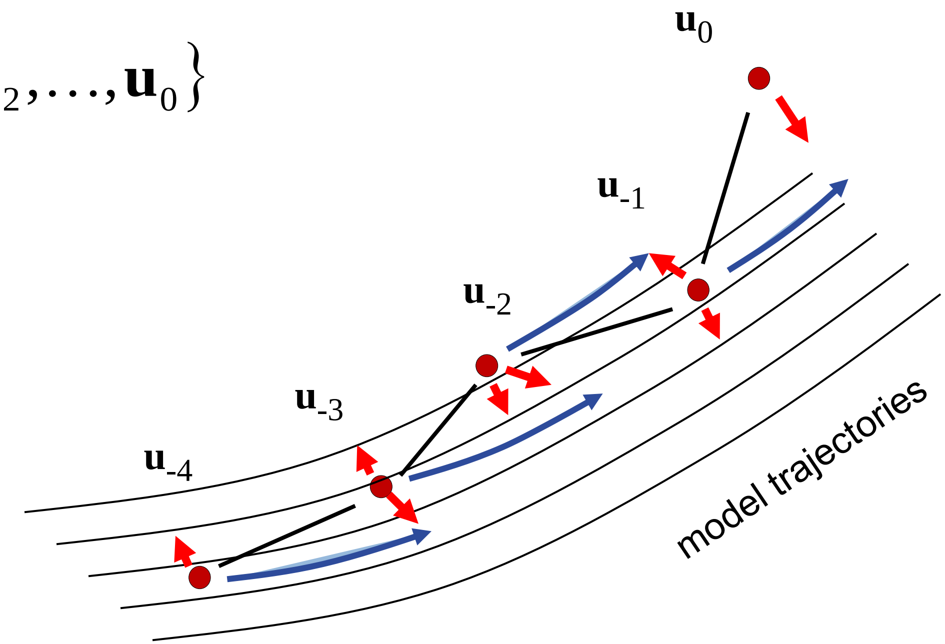
Error growth and dynamical Consistency

by Hailiang Du & Leonard A. Smith, 2013

Michael Denhard

- Pseudo-Orbit $\mathbf{U} \equiv \{\mathbf{u}_{-n+1}, \mathbf{u}_{-n+2}, \ldots, \mathbf{u}_0\}$

→ $F(\mathbf{u}_t) \neq \mathbf{u}_{t+1}$

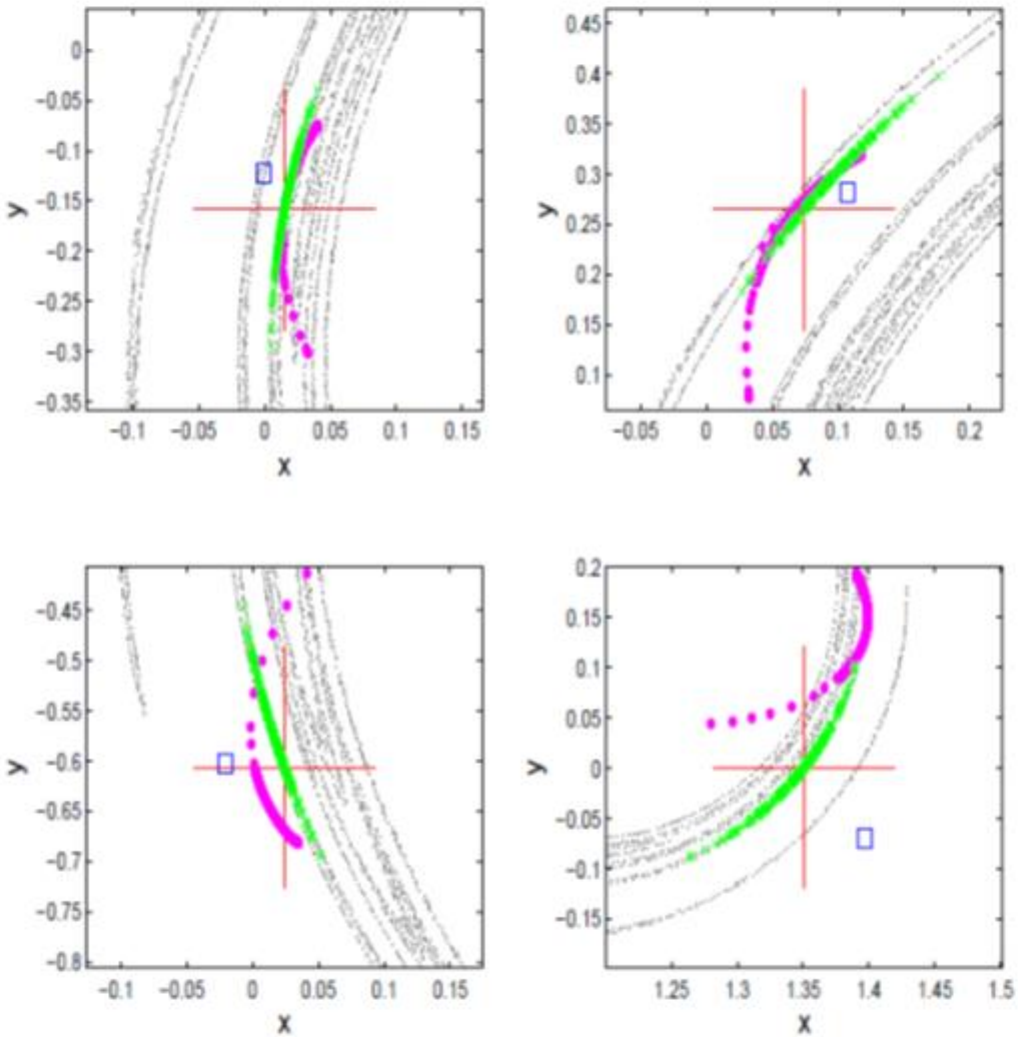


PDA Cost-Function

$$C(\mathbf{U}) = \sum \left\| F(\mathbf{u}_t) - \mathbf{u}_{t+1} \right\|^2$$

$$\frac{d\,C(\mathbf{U})}{d\,\mathbf{u}_t} = 2 \cdot \begin{cases} -(u_{t+1} - F(u_t))\, d_t F(u_t) & t = -n+1 \\ -(u_t - F(u_{t-1})) + (u_{t+1} - F(u_t))\, d_t F(u_t) & -n+1 < t < 0 \\ -(u_t - F(u_{t-1})) & t = 0 \end{cases}$$

approaches to a trajectory of the model asymptotically



Perfect Model

EnKF

PDA

□ OBS

Projection into model state space $x = g(\tilde{x})$

Observation $\mathbf{s}_t = h(g(\tilde{\mathbf{x}}_t)) + \boldsymbol{\eta}_t$

implied noise $\boldsymbol{\kappa}_t = \mathbf{s}_t - h(\mathbf{u}_t)$

$\tilde{x}$	true state
$x \in \Re^m$	projected true state
$h(\cdot)$	observation operator

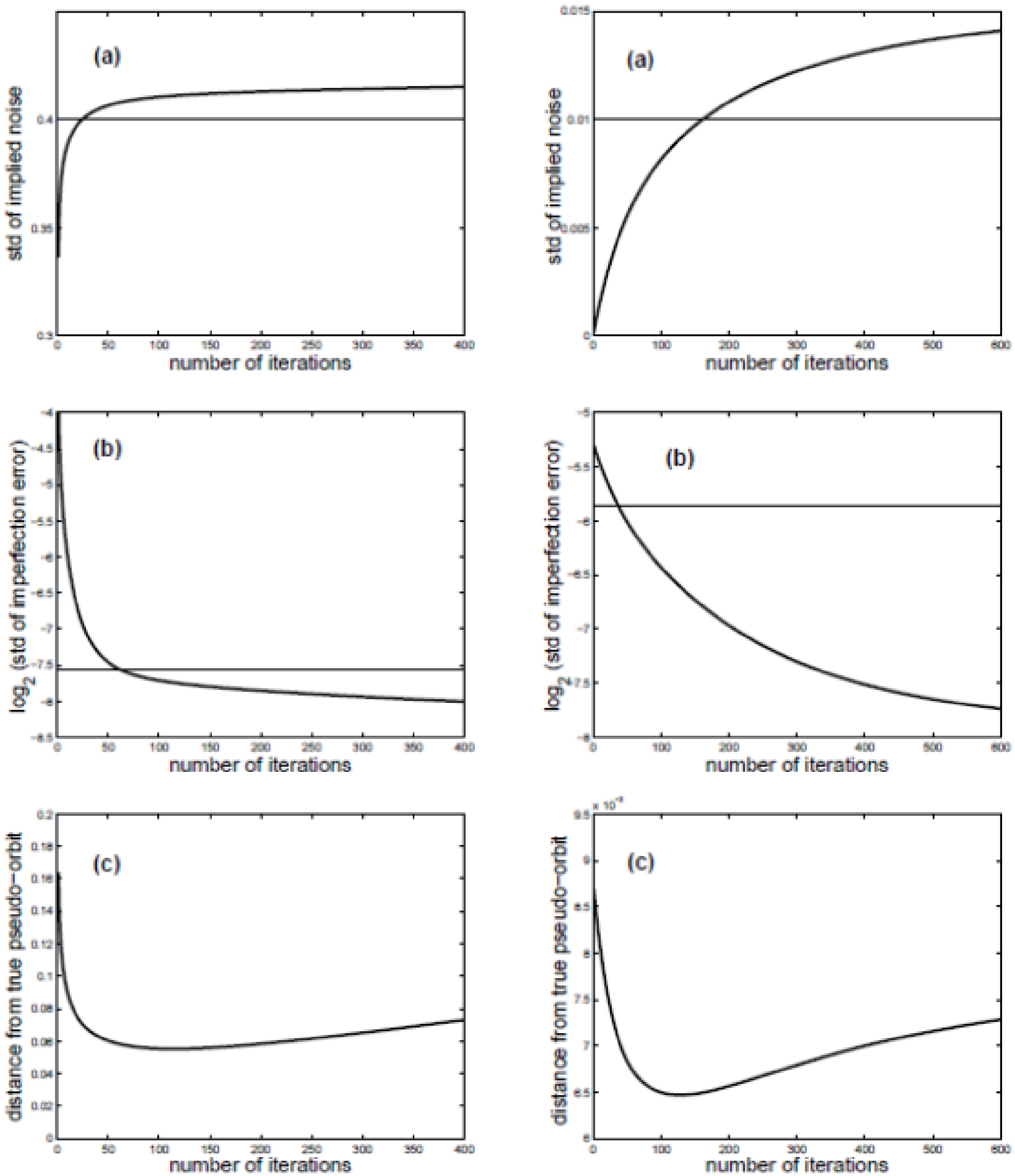
Point-wise Model Error

$$\mathbf{e}_t = g(\tilde{\mathbf{x}}_{t+1}) - F(g(\tilde{\mathbf{x}}_t))$$

Imperfection Error

$$\boldsymbol{\omega}_t = \mathbf{u}_{t+1} - F(\mathbf{u}_t)$$

PDA stopping criterion
It is desirable to obtain **pseudo-orbits** whose **implied noise** and **imperfection error** are consistent with the **observational noise** and the **point-wise model error** respectively.



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Lorenz96 (left)
Ikeda Map (right)

(a) is the standard deviation of the implied noise (the flat line is the standard deviation of the noise model)

(b) is the standard deviation of the model imperfection error (the flat line is the sample standard deviation of the point-wise model error)

(c) is the RMS distance between the pseudo-orbit and the target pseudo-orbit.

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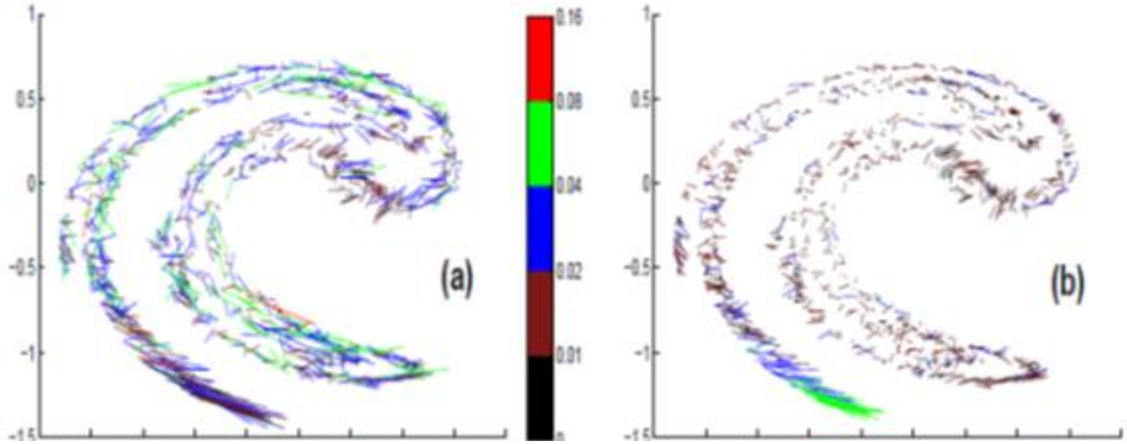
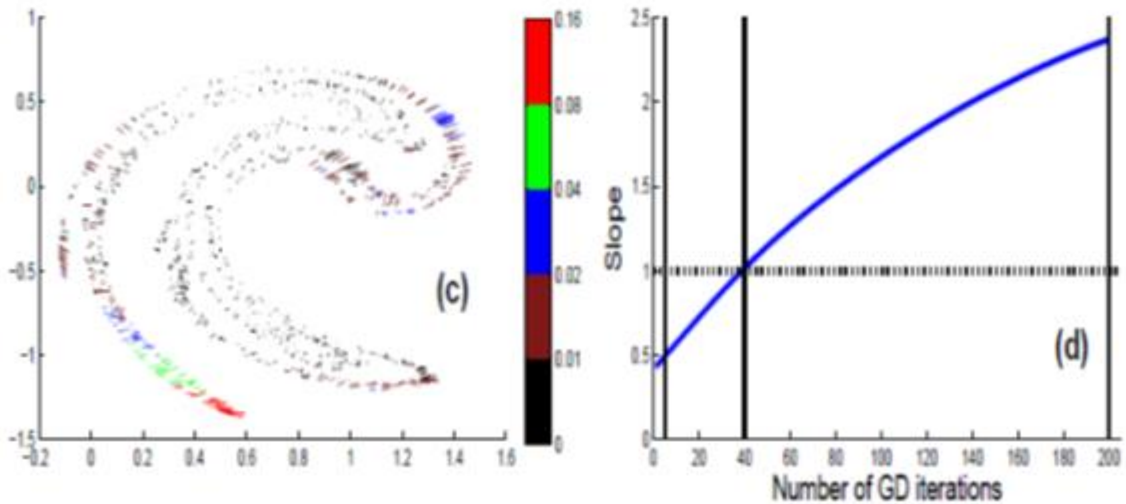


Imperfection Error (a) after 5 GD iterations, (b) after 40 GD iterations, (c) after 200 GD iterations. The color reflects the difference between the imperfection error and the corresponding pointwise model error. (d) The evolution of the slope of the best fit line relating the imperfection error and the corresponding point-wise model error



Observational noise & model inadequacy

- "PDA stopped when the standard deviation of the implied noise first exceeded the standard deviation of the observational noise.
- When the observational noise is much smaller than the point-wise model error, the latter can be well estimated by the imperfection error.
- When the observational noise is significantly bigger than the point-wise model error, the imperfection error appears more random

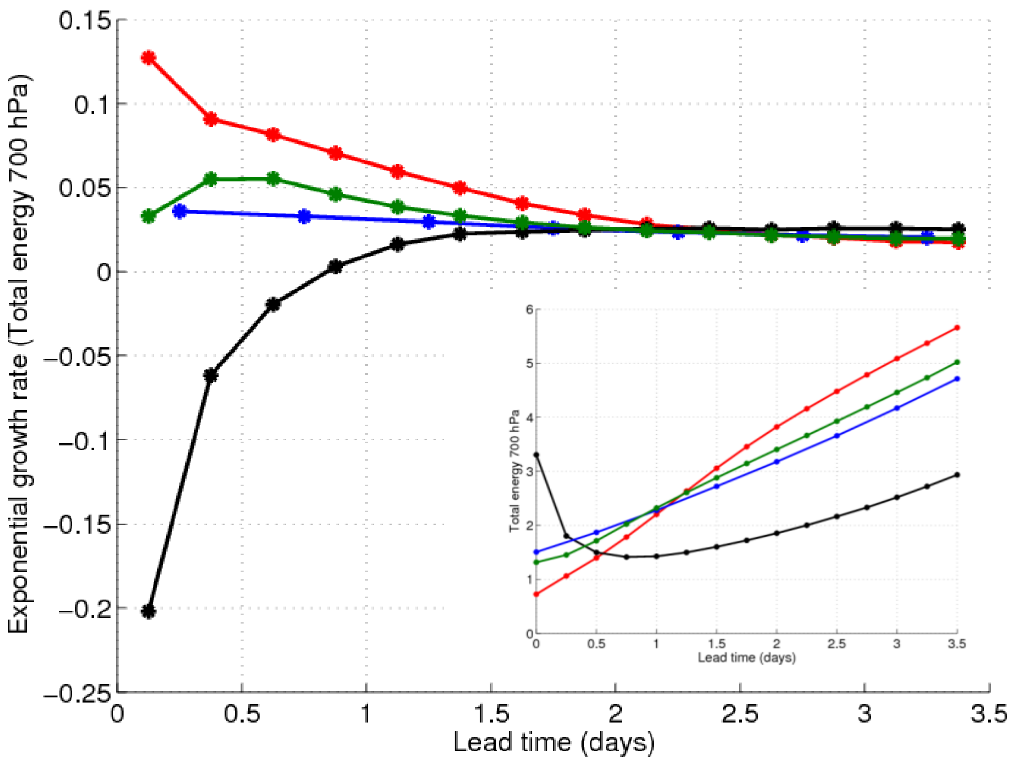
"It is stressed that state dependent model error information is an output of the proposed approach, whereas both EnKF and WC4DVAR require specifications and/or assumptions as an input."

Growth rate

$$\lambda = \frac{1}{\Delta t} \ln\left(\frac{\|\Delta x(t+\Delta t)\|}{\|\Delta x(t)\|}\right)$$

$$\Delta x = x_p - x_{true}$$

NWP-model (ECMWF)



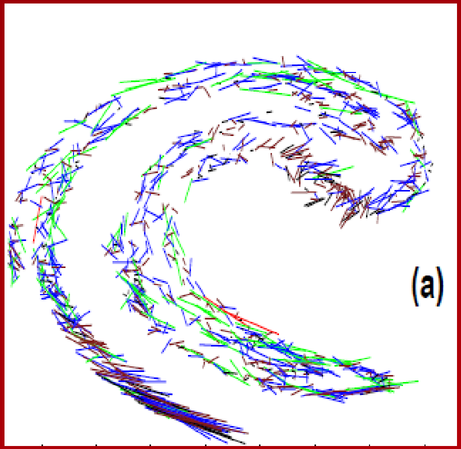
SV – red, BV – blue, Random Field Pert. – Green, Random Pert. - black

Pseudo Orbit Data Assimilation

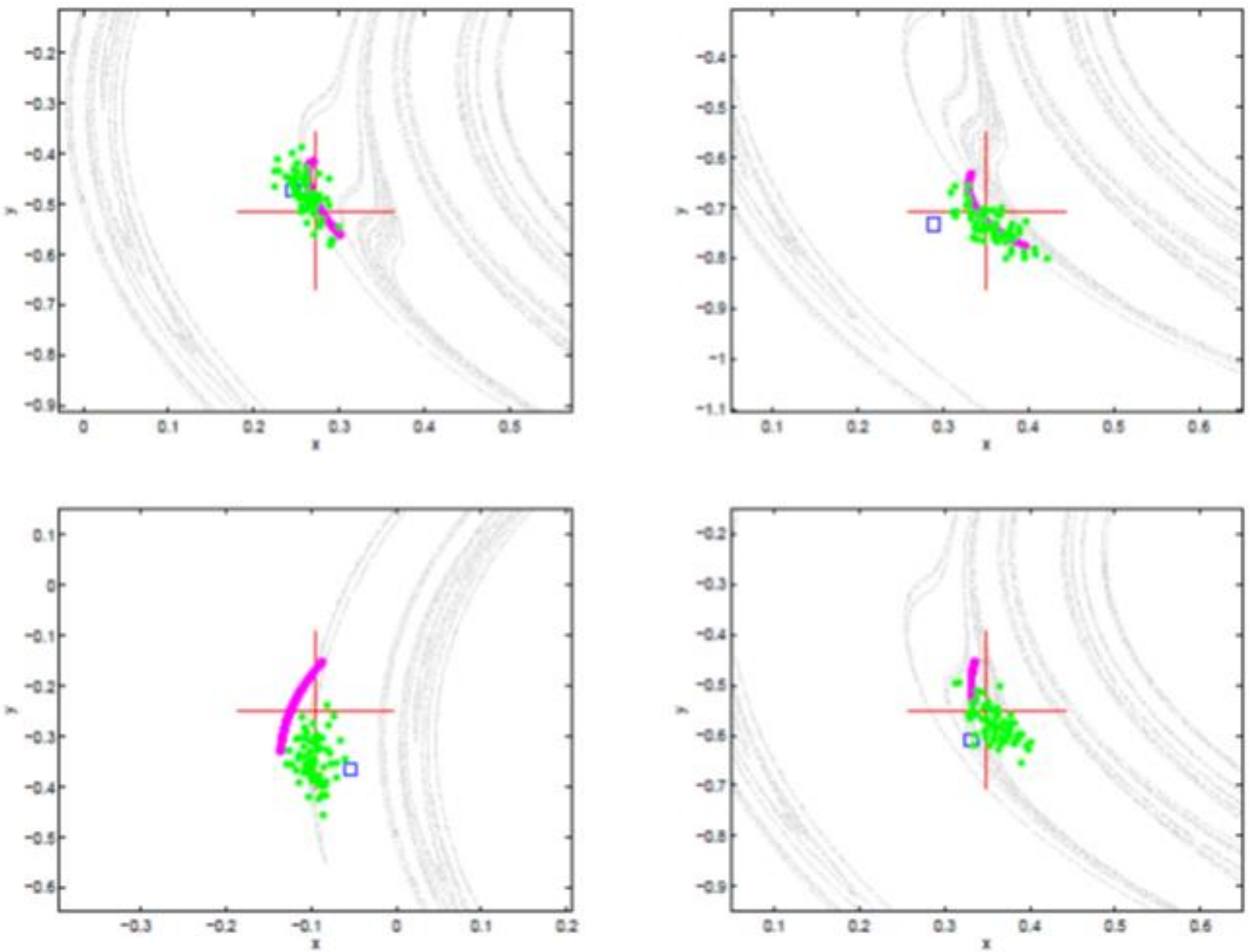
PDA cost function leads to trajectories in the model state space

PDA can be used to provide more dynamically
consistent initial states

Use it as a postprocessing tool for
EnKF - Pseudo Orbits



The following slights are optional



Imperfect Model

EnKF

PDA

□ **OBS**