COSMO-User Seminar 2014

First results of the Discontinuous Galerkin core in COSMO with moisture

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contents

budget equations of DG-COSMO

reference state

$$\begin{aligned}
\rho(x,y,z,t) &= \rho_{\text{ref}}(z) + \rho'(x,y,z,t)\\
\rho\theta_m(x,y,z,t) &= (\rho\theta_m)_{\text{ref}}(z) + (\rho\theta_m)'(x,y,z,t)\\
p(x,y,z,t) &= p_{\text{ref}}(z) + p'(x,y,z,t)
\end{aligned}$$

Euler equations with diffusion term

$$\begin{aligned}
\frac{\partial\rho'}{\partial t} &+ \nabla\cdot(\rho\mathbf{u}) &&= 0\\
\frac{\partial\rho\mathbf{u}}{\partial t} &+ \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u} + p' I_3 - \mu_m\rho\nabla\mathbf{u}) &&= -\rho' g\mathbf{k}\\
\frac{\partial(\rho\theta_m)'}{\partial t} &+ \nabla\cdot(\rho\theta_m\mathbf{u} - \mu_h\rho\nabla\theta_m) &&= s_\theta\\
\frac{\partial\rho_x}{\partial t} &+ \nabla\cdot(\rho_x\mathbf{u} - \mu_h\rho\nabla q_x) &&= s_x, \quad x\in\{v,c\}\\
\frac{\partial\rho_r}{\partial t} &+ \nabla\cdot(\rho_r\mathbf{u} - v_T\mathbf{k}) &&= s_r
\end{aligned}$$

moist potential temperature and equation of state

$$\theta_m = \frac{R_m}{R_d} T \left(\frac{p_0}{p}\right)^{\frac{R_m}{c_{\text{pml}}}} \qquad p = p_0\left(\frac{R_d}{p_0}\rho\theta_m\right)^{\frac{c_{\text{pml}}}{c_{\text{vml}}}}$$

reformulation and weak solution

prognostic var.: $\mathbf{q} = (\rho', \rho u, \rho v, \rho w, \rho\theta'_m, \rho_v, \rho_c, \rho_r)$,
primitive var.: $\mathbf{h} = (1, u, v, w, \theta_m, q_v, q_c, q_r) = \frac{\mathbf{q}+\mathbf{q}_{\text{ref}}}{\rho}$

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \cdot (\mathbf{F}(\mathbf{q}) - A(\nabla \mathbf{h})) = \mathbf{S}(\mathbf{q})$$

reformulation

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \cdot (\mathbf{F}(\mathbf{q}) - A(\boldsymbol{D})) = \mathbf{S}(\mathbf{q}) \quad \Big| \int_\Omega \cdot\, \phi \, d\Omega$$

$$\nabla \mathbf{h} = \boldsymbol{D} \quad \Big| \int_\Omega \cdot\, \phi \, d\Omega$$

weak solution

find $\mathbf{q}$ that solves the following equations

$$\int_\Omega \frac{\partial \mathbf{q}}{\partial t} \phi \, d\Omega + \int_\Omega \nabla \cdot (\mathbf{F}(\mathbf{q}) - A(\boldsymbol{D}))\phi \, d\Omega = \int_\Omega \mathbf{S}(\mathbf{q}) \, \phi \, d\Omega$$

$$\int_\Omega \nabla \mathbf{h} \, \phi \, d\Omega = \int_\Omega \boldsymbol{D} \, \phi \, d\Omega$$

for all test functions ϕ of function space V.

approximation

grid $\Omega = \bigcup_r \Omega_r$ with $\mathbf{x} \in \Omega_i \cap \Omega_j \Rightarrow \mathbf{x} \in \partial\Omega_i \cap \partial\Omega_j$

solution $\mathbf{q}|_{\Omega_r} \approx \mathbf{q}_r := \sum_{l=1}^{d} \mathbf{q}_{rl}(t)\phi_l(\mathbf{x})$, with $d < \infty$

$\langle \phi_l \mid l = 1, \ldots, d \rangle =: V_h \subset V$

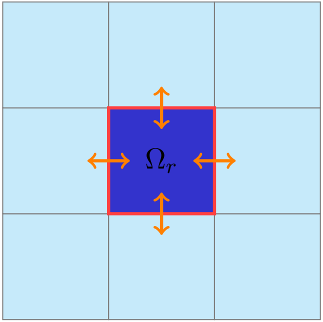
ansatz space and test space

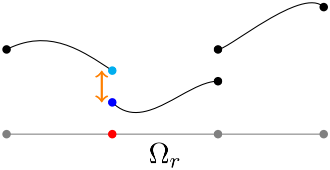
for a given degree κ (this will result in a scheme of order $\kappa + 1$) let

$$V_h = \left\langle \phi(\mathbf{x}) \;\middle|\; \phi = \prod_{i=1}^{3} P_{l_i}(x_i), \; \sum_{i=1}^{3} l_i \leq \kappa \right\rangle$$

with P_i is i-th normalised Legendre polynomial.

Discontinuous Galerkin methods (DG)





$$(\tilde{\mathbf{F}}(\mathbf{q}(\mathbf{x}_\bullet))\mathbf{n})^\star = \frac{1}{2}\left((\tilde{\mathbf{F}}(\mathbf{q}_\bullet) + \tilde{\mathbf{F}}(\mathbf{q}_\bullet))\mathbf{n} - \lambda(\mathbf{q}_\bullet - \mathbf{q}_\bullet)\right)$$

$$\int_{\Omega_r} \frac{\partial \mathbf{q}}{\partial t}\phi \, d\Omega_r = \int_{\Omega_r} (\mathbf{F}(\mathbf{q}) - A(\boldsymbol{D})) \cdot \nabla\phi \, d\Omega_r - \int_{\partial\Omega_r} ((\mathbf{F}(\mathbf{q}) - A(\boldsymbol{D})) \cdot \mathbf{n})^\star \phi \, ds + \int_{\Omega_r} \mathbf{S}(\mathbf{q})\phi \, d\Omega_r$$

$$\int_{\Omega_r} D_{ij}\,\phi \, d\Omega_r = \int_{\Omega_r} p_i \frac{\partial \phi}{\partial j} \, d\Omega_r - \int_{\partial\Omega_r} (h_i n_j)^\star \phi \, ds$$

the spacial DG discretisation results in a coupled system of *ordinary differential equations*:

$$\frac{\partial \mathbf{q}}{\partial t} = L(t, \mathbf{q}).$$

integrable with Runge-Kutta, predictor-corrector schemes, ...

time step / CFL

for explicite stable time integration holds:

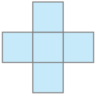
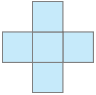
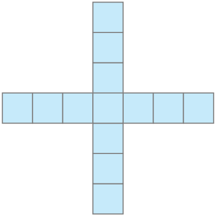
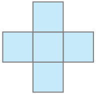
$$(|\mathbf{u}| + c)\frac{\Delta t}{\Delta x} < \frac{1}{2\kappa + 1}.$$

example

1 DoFs/$(2.8\,\text{km})^2$ horizontal, $|\mathbf{u}| + c = 350\,\frac{\text{m}}{\text{s}}$,

COSMO-DE fast waves,	$\Delta t = 4\frac{1}{6}$ s,	$\Delta x = 2.8$ km,
$\kappa = 1$,	$\Delta t \le 4.2$ s,	$\Delta x = 4.4$ km,
$\kappa = 2$,	$\Delta t \le 3.4$ s,	$\Delta x = 6.0$ km,
$\kappa = 3$,	$\Delta t \le 3.2$ s,	$\Delta x = 7.6$ km,
$\kappa = 3$, (full tensor product)	$\Delta t \le 3.6$ s,	$\Delta x = 11.2$ km

stencil

	FD	(Compact-) DG
method of 2. order		
method of 5. order		

the stencil of a DG method is independent of the order.

background atmosphere in 2d channel

length $L = 300\,\mathrm{km}$, height $H = 10\,\mathrm{km}$,
isothermal, hydrostatic $\left(\frac{\partial p}{\partial z} = -\rho g\right)$, constant Brunt-Väisälä frequency

$$T_0(z) = T_{00} = 250\,\mathrm{K}, \quad p_0(z) = p_{00}\, e^{-\delta z}, \quad \rho_0(z) = \rho_{00}\, e^{-\delta z}, \quad \delta = \frac{g}{R\, T_{00}}$$

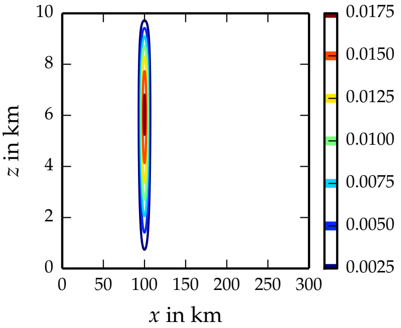
initial state

$$T'(\mathbf{x}) = \Delta T e^{-\frac{(x-x_c)^2}{d^2}} e^{\frac{1}{2}\delta z} \sin\left(\pi \frac{z}{H}\right),$$

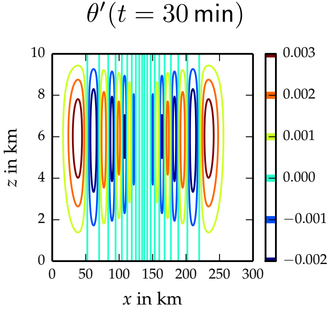
$$\Delta T = 0,01\,\mathrm{K},$$

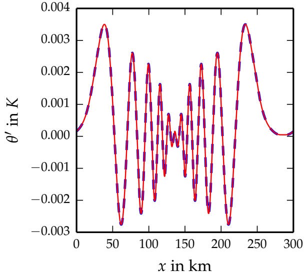
$$d = 5\,\mathrm{km},$$

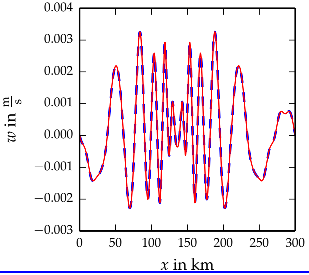
$$\mathbf{u} = \begin{pmatrix} 20\,\frac{\mathrm{m}}{\mathrm{s}} \\ 0\,\frac{\mathrm{m}}{\mathrm{s}} \end{pmatrix}$$



Baldauf/Brdar 2013

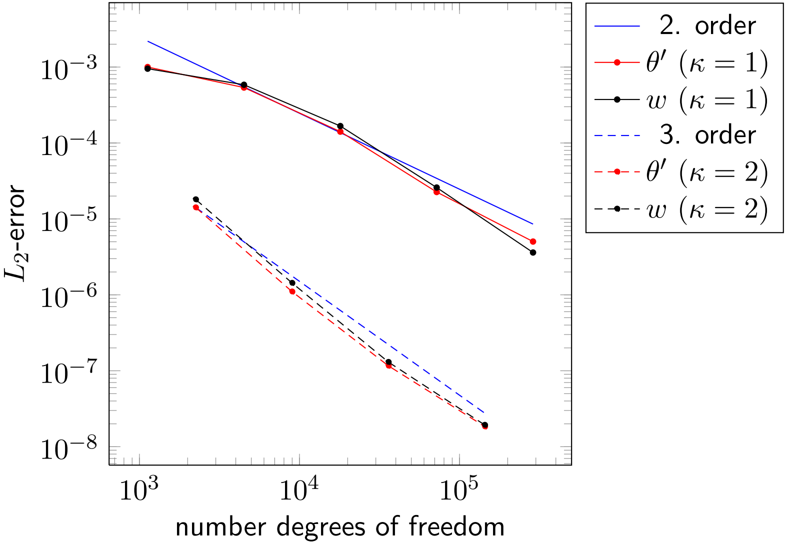






linearised solution dashed blue lines
DG-COSMO $\kappa = 3$, $\Delta x = 500\,\mathrm{m}$, red line

convergence (Baldauf/Brdar 2013)



scalability and speedup

test case for scalability

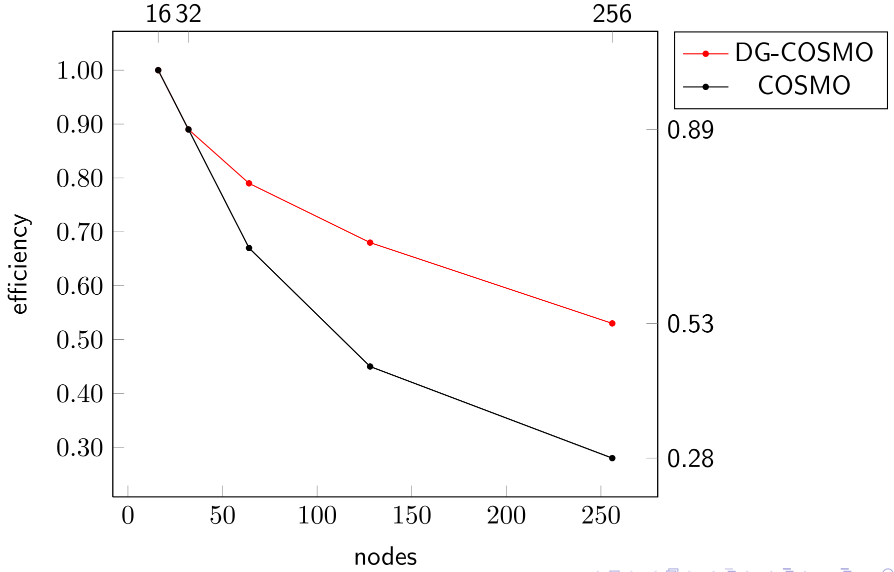
DG-COSMO	COSMO
three dimensional domain	
hydrostatic balanced background	
initial warm bubble	
$t = 0\,\mathrm{s}, \ldots, 18\,\mathrm{s}$	
16 000 000 degrees of freedom	
$\Delta t = 0,05\,\mathrm{s}$	
RK3, $\kappa = 3$, full tensor product	RK3, adv 5. Order
$100 \times 100 \times 25$ elements	$400 \times 400 \times 100$ elements

speedup and efficiency

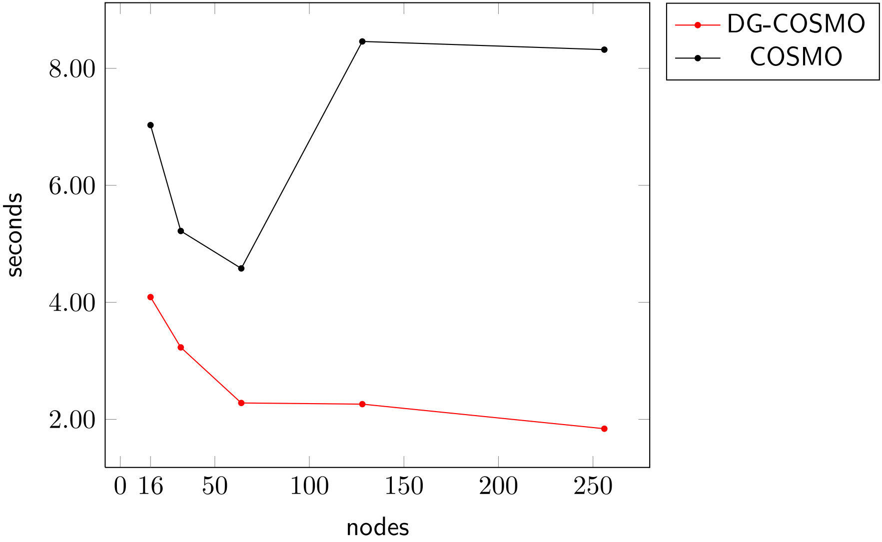
Let L_n be the runtime of program P on n CPUs and L'_m of program P' on m CPUs. The speedup $S(L'_m, L_n)$ of L'_m w.r.t L_n and the efficiency $E_m(L_n)$ of L_n w. r. t. m CPUs are

$$S(L'_m, L_n) = \frac{L_n}{L'_m}, \qquad E_m(L_n) = \frac{m}{n} \cdot \frac{L_m}{L_n}.$$

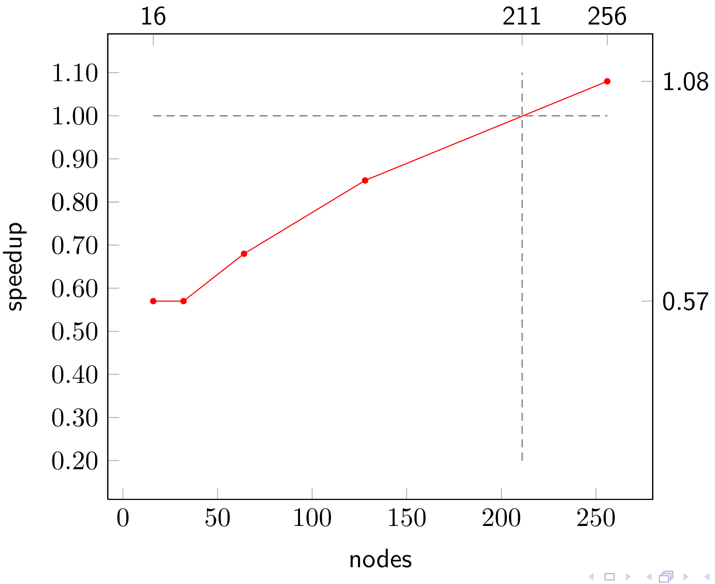
strong scalability (Cray XC30; warm 3d-bubble)



average communication time per node



speedup of DG-COSMO w.r.t. COSMO



Results of the moist atmosphere

physics coupled

- isochor saturation adjustment of Ulrich Blahak (DWD)
- Kessler-scheme for warm rain

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

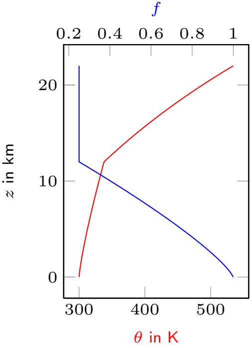
background atmosphere

Profile for θ and relative humidity f

$$\theta(z) = \begin{cases} \theta_{00} + (\theta_{tr} - \theta_{00}) \left(\frac{z}{z_{tr}}\right)^{\frac{5}{4}}, & z \leq z_{tr}, \\ \theta_{tr}\, e^{\frac{g}{c_p T_{tr}}(z - z_{tr})}, & z > z_{tr}, \end{cases}$$

$$f(z) = \begin{cases} 1 - \frac{3}{4} \left(\frac{z}{z_{tr}}\right)^{\frac{5}{4}}, & z \leq z_{tr}, \\ \frac{1}{4}, & z > z_{tr}, \end{cases}$$

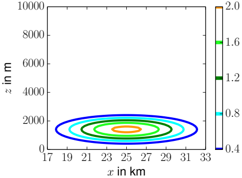
where $z_{tr} = 12\,\text{km}$, $\theta_{00} = 300\,\text{K}$ $\theta_{tr} = 338\,\text{K}$ $T_{tr} = 213\,\text{K}$,
q_v is clipped to maximum 0.014
constant viscosity $\mu = 50\,\frac{\text{m}^2}{\text{s}}$



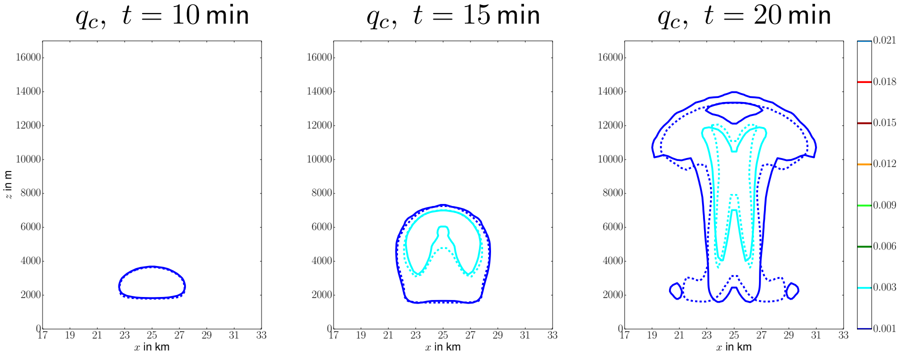
initial state

$$\Delta T = \begin{cases} d_T \cdot \cos(\frac{\pi}{2} L)^2, & L \leq 1, \\ 0, & L > 1, \end{cases}$$

where $L = \sqrt{\left(\frac{x - x_c}{r_x}\right)^2 + \left(\frac{y - y_c}{r_y}\right)^2 + \left(\frac{z - z_c}{r_z}\right)^2}$, with $d_T = 2\,\text{K}$,
$r_x = r_y = 10\,\text{km}$, $r_z = 1\,400\,\text{m}$. $z_c = 1\,400\,\text{m}$

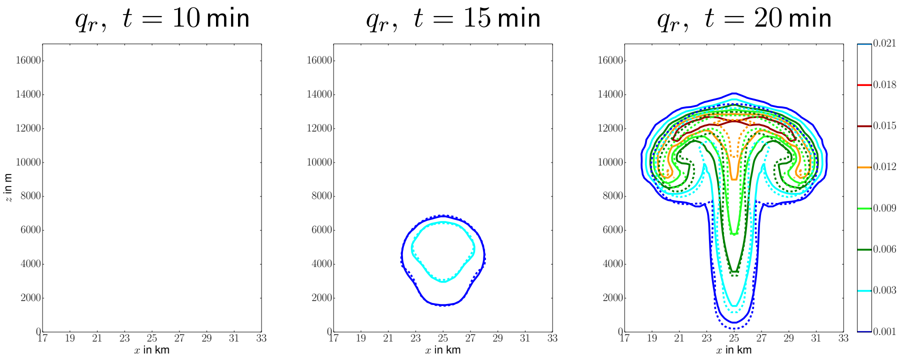


WK82 cloud water



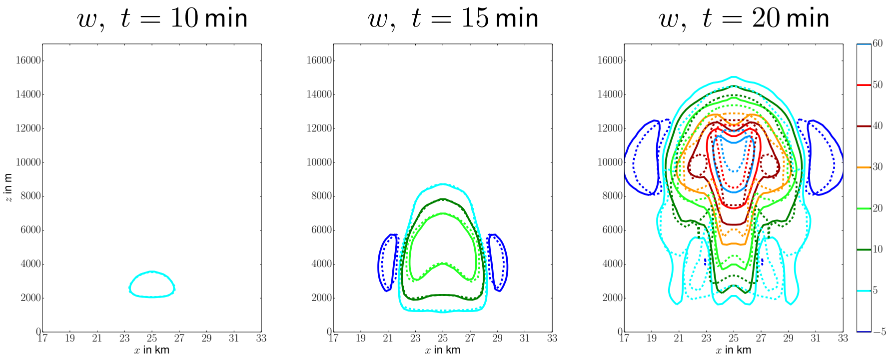
COSMO dashed lines, DGCOSMO solid lines

WK82 rain water



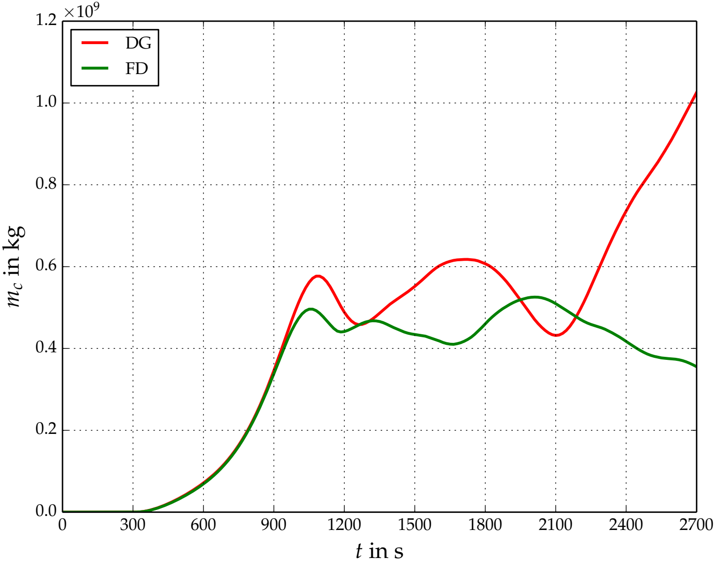
COSMO dashed lines, DGCOSMO solid lines

WK82 vertical velocity



COSMO dashed lines, DGCOSMO solid lines

WK82 total cloud water



summary and outlook

DG-COSMO ...

- ... brings a higher order scheme with local conservation properties for the dynamics
- ... has the potential to use NWP high efficiently on exa-scale computers
- ... is ready for the first semi-real simulations

next steps

- first simulation with real data
- semi-implicit or HEVI scheme (**H**orizontal **E**xplicit, **V**ertical **I**mplicit)
  vertical grid spacing does no longer restrict the time step
- coupling of missing physics

acknowledgement

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literature

[1] Michael Baldauf and Slavko Brdar. *An analytic solution for linear gravity waves in a channel as a test for numerical models using the non-hydrostatic, compressible Euler equations*. *Quarterly Journal of the Royal Meteorological Society*. () 2013.

[2] M. L. Weisman and J. B. Klemp. *The Dependence of Numerically Simulated Convective Storms on Vertical Wind Shear and Buoyancy*. *Monthly Weather Review*. 110. () 1982.