Kinetic Energy spectra of COSMO model using Higher Order Spatial schemes

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Outline

- Introduction
- Higher order spatial schemes in COSMO model
- Kinetic Energy spectrum
- Conclusions

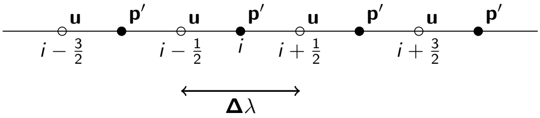
Introduction

Why Kinetic energy spectrum?

- Deviations from observed energy cascade indicates unphysical scale interaction which can be attributed to violation of conservation principles.
- Non-linear instability in atmospheric prediction models has been attributed to unphysical buildup of kinetic energy at small-scales

Higher order spatial schemes in COSMO model

c-grid:



Zonal Numerical operators

Centered finite differencing:

$$\overline{\psi}^{n\lambda} = \frac{\psi_{i+n/2} + \psi_{i-n/2}}{2}, \qquad \delta_{n\lambda}\psi = \frac{\psi_{i+n/2} - \psi_{i-n/2}}{n\Delta\lambda}, \qquad \forall\ i, n \in \mathbb{N}$$

Higher order spatial schemes in COSMO model

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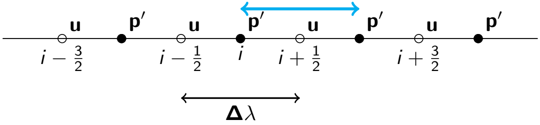

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- Second order operators: (for $\psi = p'$)

$$\overline{\psi}^{O2} := \overline{\psi}^{\lambda}, \quad \delta^{O2}(\psi) := \delta_{\lambda}\psi$$

Higher order spatial schemes in COSMO model

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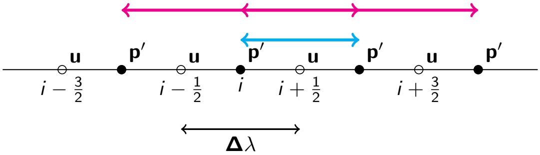

Zonal Numerical operators

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- **Second order operators: (for $\psi = p'$)**

$$\overline{\psi}^{O2} := \overline{\psi}^{\lambda}, \quad \delta^{O2}(\psi) := \delta_{\lambda}\psi$$

- **Fourth order operators: (for $\psi = p'$)**

$$\overline{\psi}^{O4} := \frac{9}{8}\overline{\psi}^{\lambda} - \frac{1}{8}\overline{\psi}^{3\lambda}, \quad \delta^{O4}(\psi) := \frac{9}{8}\delta_{\lambda}\psi - \frac{1}{8}\delta_{3\lambda}\psi$$

Higher order spatial schemes in COSMO model

u **momentum in terrain following coordinates** $(\lambda, \phi, \zeta(\lambda, \phi, z))$**:**

$$\frac{\partial u}{\partial t} = -\frac{1}{r\cos\phi}\left(u\frac{\partial u}{\partial \lambda} + v\cos\phi\frac{\partial u}{\partial \phi}\right) - \dot{\zeta}\frac{\partial u}{\partial \zeta} - \frac{1}{\rho}\frac{1}{r\cos\phi}\left(\frac{\partial p'}{\partial \lambda} + \frac{\partial \zeta}{\partial \lambda}\frac{\partial p'}{\partial \zeta}\right) + Damp$$

contravariant vertical velocity

$$\dot{\zeta} = \frac{1}{r\cos\phi}\left(u\frac{\partial z}{\partial \lambda} + v\cos\phi\frac{\partial z}{\partial \phi}\right) - w\frac{\partial \zeta}{\partial z}$$

artificial damping

$$Damp = \left\{\beta_1\alpha_4\nabla_\lambda^4 u + \frac{1}{\rho}\frac{1}{r\cos\phi}\left(\frac{\partial \alpha_{div}^h \rho D}{\partial \lambda} + \frac{\partial \zeta}{\partial \lambda}\frac{\partial \alpha_{div}^h \rho D}{\partial \zeta}\right)\right\}, D = div(u, v, w)$$

Higher order spatial schemes in COSMO model

u momentum in terrain following coordinates $(\lambda, \phi, \zeta(\lambda, \phi, z))$:
Zonal derivatives

$$\frac{\partial u}{\partial t} = -\frac{1}{r\cos\phi}\left(\underline{u\frac{\partial u}{\partial \lambda}} + v\cos\phi\frac{\partial u}{\partial \phi}\right) - \dot{\zeta}\frac{\partial u}{\partial \zeta} - \frac{1}{\rho}\frac{1}{r\cos\phi}\left(\underline{\frac{\partial p'}{\partial \lambda} + \frac{\partial \zeta}{\partial \lambda}}\frac{\partial p'}{\partial \zeta}\right) + Damp$$

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Higher order spatial schemes in COSMO model

Discrete Zonal Advection: $u\frac{\partial u}{\partial \lambda}$ **(similarly** $u\frac{\partial z}{\partial \lambda}$**)**

$$C3 := u\left(\frac{4}{3}\delta_{2\lambda}u - \frac{1}{3}\delta_{4\lambda}u + \frac{(\Delta\lambda)^3}{12}\frac{\partial^4 u}{\partial \lambda^4}\right)$$

$$N4 := u\left(\frac{4}{3}\delta_{2\lambda}u - \frac{1}{3}\delta_{4\lambda}u\right)$$

$$S4 := \frac{9}{8}\overline{\overline{u}^{O4}\delta_{\lambda}u}^{\lambda} - \frac{1}{8}\overline{\overline{u}^{O4}\delta_{3\lambda}u}^{3\lambda}$$

$$= \underbrace{\left[\frac{9}{8}\delta_{\lambda}(\overline{u}^{O4}\overline{u}^{\lambda}) - \frac{1}{8}\delta_{3\lambda}(\overline{u}^{O4}\overline{u}^{3\lambda})\right]}_{\equiv \frac{\mathbf{F_{i+n/2}-F_{i-n/2}}}{\mathbf{n\Delta\lambda}}} - u\left[\frac{9}{8}\delta_{\lambda}(\overline{u}^{O4}) - \frac{1}{8}\delta_{3\lambda}(\overline{u}^{O4})\right]$$

Higher order spatial schemes in COSMO model

Discrete Zonal Advection: $u\frac{\partial u}{\partial \lambda}$ **(similarly** $u\frac{\partial z}{\partial \lambda}$**)**

dissipative term

divergence form: conservative a *priori*

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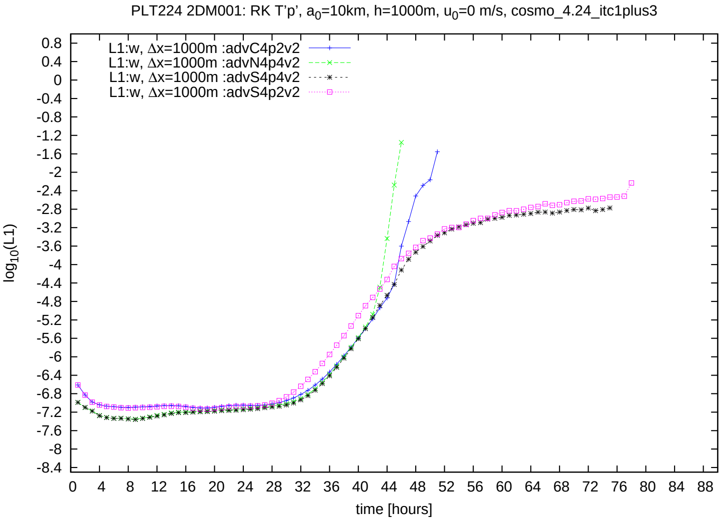
Properties of different spatial schemes in COSMO model

Schemes:	Implicit diffusion	KE conservation	Explicit diff. coefficient
$C3p2v2d0.00$:	Yes	No	0.00
$C3p2v2d0.25$:	Yes	No	0.25
$N4p4v2d0.25$:	No	No	0.25
$S4p4v2d0.00$:	No	Yes[1]	0.00
$S4p4v2d0.25$:	No	No	0.25

[1]If the continuity equation is satisfied.

Numerical Tests

Atmosphere resting on a small hill: $h_{max} = 3000m$



Real boundary condition simulation

Configuration for the Real case simulation:

- The current standard evaluation configuration approved by the COSMO-CLM Community (Keuler et al, (2012)) is used with; Model version 4.24, new fast waves solver and new schemes

- Domain: COSMO_EU
- Forcing data: ERA-interim reanalyses
- $\Delta x = 18.3km$,
- $ke\_tot = 40$
- $\Delta t = 150s$

Real boundary condition simulation

Model run time at $\Delta t = 150s$:

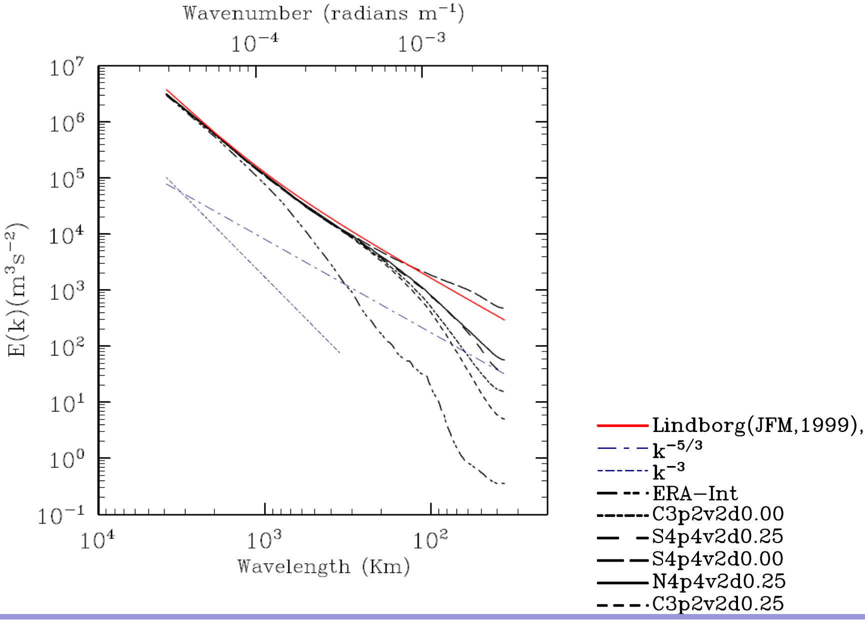
Schemes:	Run time
$N4p4v2d0.00$:	Upto 2nd month
$N4p4v2d0.25$:	Beyond 3 months
$S4p4v2d0.00$:	Beyond 3 months
$S4p2v2d0.00$:	Upto 2nd month ?[1]

[1]Inconsistency in the discretisation of the advection and pressure gradient terms

Kinetic energy spectrum

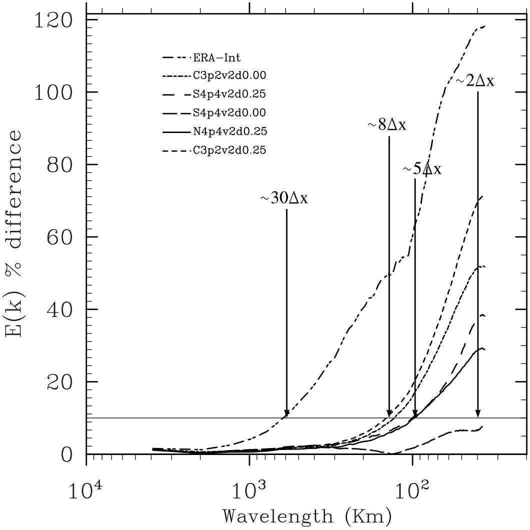
- The model output is detrended as proposed by (Errico, MWR (1985)
- The spectra are then calculated using fast fourier transformation technique(FFT).
- The calculated spectra are then compared to the observation results of (Lindborg, JFM (1999))

Mean Kinetic Energy spectrum (700 – 400hpa)



Kinetic energy spectrum

Effective resolution: $(COSMO - OBS)/OBS < 10\%$



Conclusions

- Fourth order scheme $S4p4v2d0.00$ is as stable as the third order upwind scheme $C3p2v2d0.25$
- Fourth order schemes have high effective resolution compared to the third order upwind scheme.
- Implicit (present in the third order upwind scheme) and explicit horizontal diffusion reduces effective resolution of the model.
- Fourth order scheme $S4p4v2d0.00$ exhibits an effective resolution which is close to $2\Delta\lambda$

THANK YOU...

For climatological results from the 5-yr climate simulation using the new schemes, please see poster " Impact of Higher Order horizontal spatial discretisation of Euler equations in the COSMO model on regional climate over Europe, Ogaja, et al."