Climatological relevance of higher order horizontal discretisation of the Euler equations

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Introduction

We implemented a 4^{th} order horizontal disretisation of the Euler equations in the COSMO model. Two types of the advection term discretisation (Nn, Sn) with discretisation error convergence of order n have been implemented. The Nn advection is calculated at the grid point of the advected quantity as for all COSMO advection schemes (Cn). The Sn advection is calculated at the center of the advecting velocity and the advected quantity. This allows a symmetric formulation of all advection components and conserves the 1^{st} and 2^{nd} moments of the advected quantity if the continuity equation is satisfied *(Morinishi,1999)*.

The advection schemes can be combined with 2^{nd} or 4^{th} order pressure gradient term discretisation, referred to as (p2, p4) respectively. The metric terms and coefficients are discretised 4^{th} order as well. The S4p4 discretisation has better stability properties, which allows to reduce the coefficient h of the horizontal diffusion d, referred as (dh).

We investigated the relevance of the discretisation error convergence, type of advection and strength of horizontal diffusion for the regional climate over Europe. We used the recommended configuration (0.165°) of the CLM Community (Keuler et al., 2012) adapted to model version 4.24 with the new fast wave solver (Baldauf, 2013) and simulated the period 1979-1983 for the schemes defined in Table 1.

Numerical Schemes

2^{nd} order zonal interpolation and differencing

$$\overline{\psi}^{n\lambda} = \tfrac{1}{2}\left(\psi_{i-n/2} + \psi_{i+n/2}\right) \qquad \delta_{n\lambda}\psi = \frac{\psi_{i+n/2}-\psi_{i-n/2}}{n\Delta\lambda}$$

4^{th} order zonal interpolation and differencing

$$\overline{\psi}^{O4,\lambda} := \frac{9}{8}\overline{\psi}^{\lambda} - \frac{1}{8}\overline{\psi}^{3\lambda}, \quad \delta^{O4,\lambda}\psi := \frac{9}{8}\delta_{\lambda}\psi - \frac{1}{8}\delta_{3\lambda}\psi$$

$$\overline{\psi}^{O4,2\lambda} := \frac{4}{3}\overline{\psi}^{2\lambda} - \frac{1}{3}\overline{\psi}^{4\lambda}, \delta^{O4,2\lambda}\psi := \frac{4}{3}\delta_{2\lambda}\psi - \frac{1}{3}\delta_{4\lambda}\psi$$

Zonal Advection: $u\frac{\partial u}{\partial \lambda} + v\frac{\partial u}{\partial \phi}$

$$N4(u) := u\delta^{O4,2\lambda}u + \overline{\overline{v}^{O4,\lambda}}^{O4,\phi}\delta^{O4,2\phi}u$$

$$S4(u) := \overline{\overline{u}^{O4,\lambda}\delta^{O4,\lambda}u}^{O4,\lambda} + \overline{\overline{v}^{O4,\lambda}\delta^{O4,\phi}u}^{O4,\phi}$$

Overview

Table 1: Horizontal schemes and Simulation IDs.

Scheme:	adv	pres	hor.dif.	Simul. ID
C3p2v2d0.25	C3	O2	*d0.25*	RTC002
N4p4v2d0.25	N4	O4	*d0.25*	RTC004
S4p4v2d0.25	S4	O4	*d0.25*	RTC016
S4p4v2d0.00	S4	O4	*d0.00*	RTC012

References

[1] Y. Morinishi, T. Lund, O.V. Vasilyev, P. Moin: *Fully conservative higher order finite difference schemes for incompressible flow*, J. Comput. Phys. 143 (1998) 90-124

[2] Klaus Keuler, Kai Radtke, Goran Georgievski: *Summary of evaluation results for COSMO-CLM version 4.8_clm13 (clm17): Comparison of three different configurations over Europe driven by ECMWF reanalysis data ERA40 for the period 1979-2000*, Evaluation Report, BTU Cottbus, April 2012, www.clm-community.eu

[3] Michael Baldauf: *A new fast-waves solver for the Runge-Kutta dynamical core*, Technical Report 21, COSMO , April 2013, www.cosmo-model.org

Zonal spectra

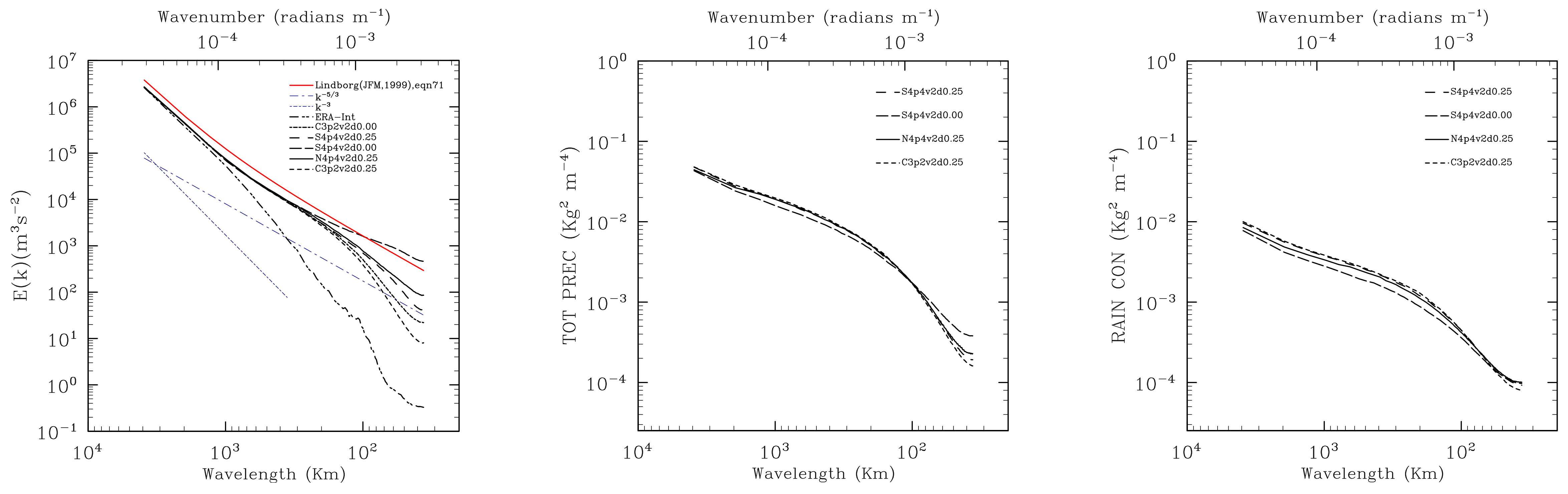


Fig.1: Power spectra of zonal kinetic energy (left) total precipitation (center) and convective rain (right) The figures exhibit the mean upper troposphere JJA power spectra of COSMO-CLM simulations for the schemes given in Table 1. Additionally the kinetic energy spectra derived from observations (Lindborg), ERA interim and for the scheme C3p2d0.00 is shown together with the theoretical spectra $k^{-5/3}$ and k^{-3}. The mean is taken from 500 to 300 hPa and the latitudes of the domain (without relaxation zone). The S4p4d0.00 scheme exhibits approximately the observed kinetic energy spectrum and a reduced summer convective rain for scales longer than 100 km. All 4th order schemes show higher convective rain amounts at small scales.

2m Temperature

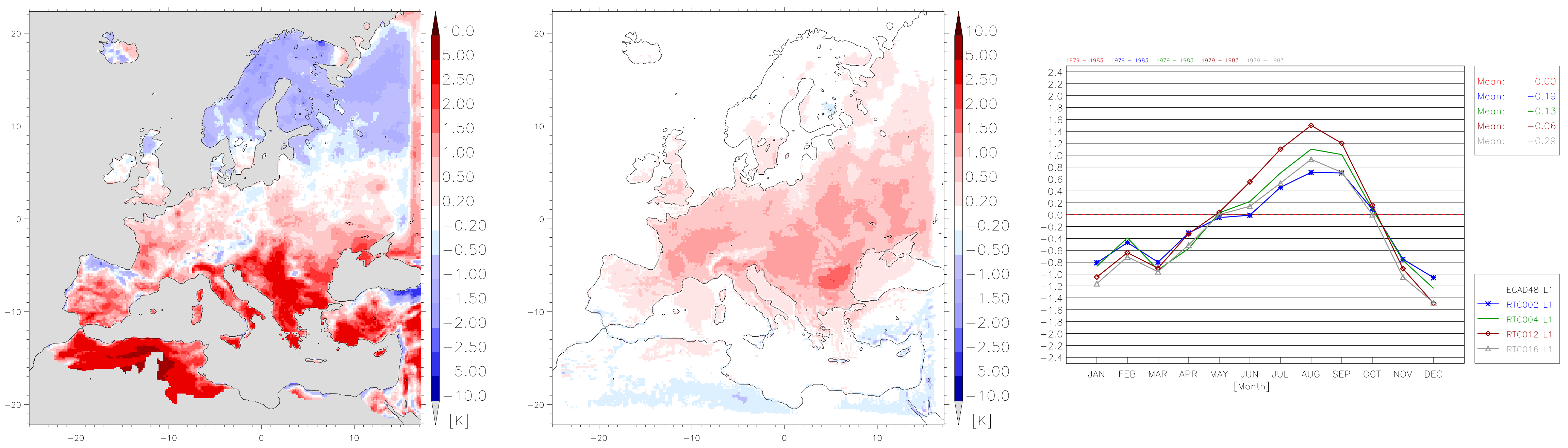
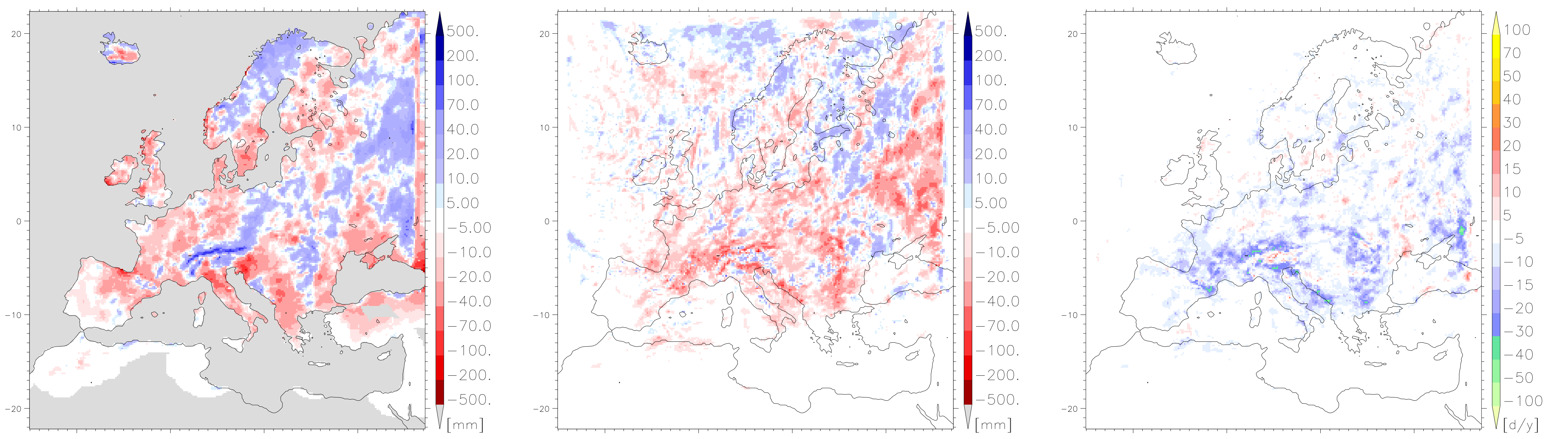


Fig.2: 2m Temperature differences, August 79-83: C3p2d0.25-ECAD (left), S4p4d0.00-C3p2d0.25 (center) and annual cycle in great Alpine region (right). The results exhibit a significant temperature decrease in Scandinavia and an increase in central to southern Europe of 0.5-1 K in summer for the 4th order symmetric advection scheme without horizontal diffusion.

Precipitation



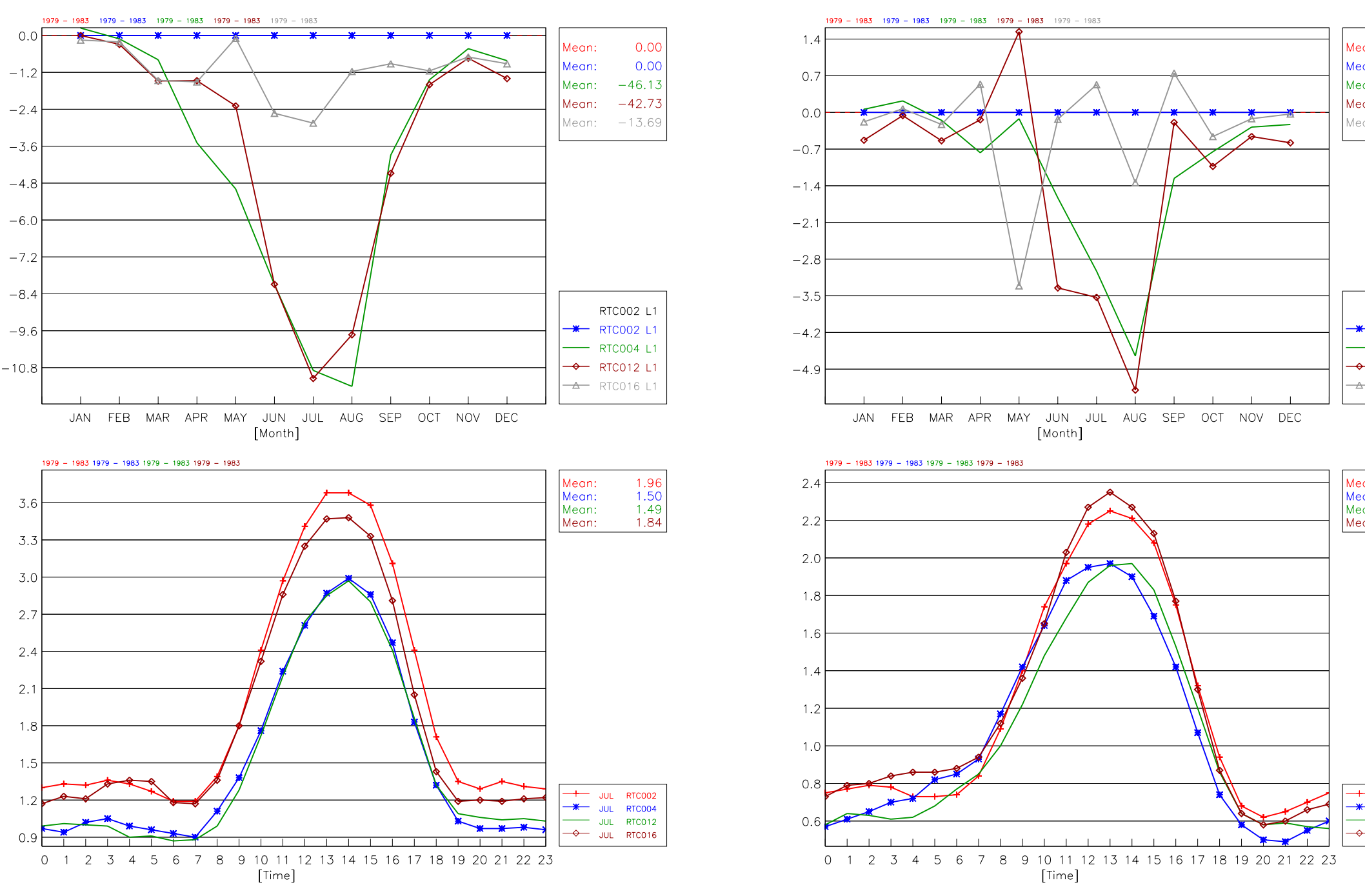


Fig.3 (top panel): Precipitation sum differences in August 79-83: C3p2d0.25-ECAD for TOT_PREC (left), S4p4d0.00-C3p2d0.25 for TOT_PREC (center) and PREC_CON (right) The results exhibit a significant decrease in summer precipitation, in particular the convective precipitation of 20-50 mm in central to southern Europe using the 4th order symmetric advection scheme without horizontal diffusion.

Fig 4 (bottom panel): Annual differences (top) and diurnal cycle (bottom) of convective precipitation in the great Alpine region (left) and Germany (right). A significant decrease for S4p4d0.25 and S4p4d0.00 was found in summer months and a 1h shift of convection towards afternoon for S4p4d0.00 in e.g. the region of Germany.

Summary

The S4p4d0.00 scheme modifies significantly the regional climate in Europe at standard resolution of 18km, in particular in summer. The summer warm bias is increased due to reduction of convective precipitation amounts.

The developments exhibit promising results with respect to model physics and dynamics. The 4th order improves the effective resolution and model stability (not shown). The slight overestimation of the kinetic energy at small scales is a known feature of 4th order schemes. The symmetric advection allows to neglect the artificial horizontal diffusion. This increases the effective resolution of model dynamics close to the theoretical limit.

The convection appears more stochastic and at later afternoon and exhibits the relation of dynamics and subgridscale parameterisations.