The Plant-Craig stochastic convection scheme: How it works and some examples of its application

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Thanks to: Bob Plant, George Craig, Günther Zängl, Warren Tennant, Christian Keil
Stochastic Physics Week
1. Overview of the Plant-Craig stochastic convection scheme

2. Scale-adaptivity of the Plant-Craig scheme in comparison with deterministic schemes, Part I
   - ICON Aquaplanet
   - Effects of the input averaging

3. Verification in MOGREPS

4. Scale-adaptivity of the Plant-Craig scheme in comparison with deterministic schemes, Part II
   - Convective equilibrium experiment
Variability of convective response to a given large-scale forcing (Convective Equilibrium Experiment)

Taken from Plant & Craig, JAS (2008)
Plant-Craig scheme methodology (I)

- Average $u, v, T, q, q_l, q_i$ over a region large enough to contain many clouds, but small enough that it is representative of the current grid box.
- Use this as input to the Kain-Fritsch trigger scheme.

If convection is initiated:

- Use the Kain-Fritsch plume model, extended to allow for an ensemble of plumes, to calculate vertical properties.
  - The plumes are weighted by $p(m)dm = \frac{1}{\langle m \rangle} e^{-m/\langle m \rangle} dm$.
- The model allows for updraft/downdraft pairs with entrainment and detrainment.
- Entrainment decreases with increasing mass flux.
- The total required mass flux is obtained by scaling so that 90% of cape is removed within the closure time scale. This yields the ensemble mean mass flux $\langle M \rangle$. The ensemble mean mass flux per cloud $\langle m \rangle$ is taken as a ‘fundamental constant’.
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Plant-Craig scheme methodology (II)

- Use $\langle M \rangle$ and $\langle m \rangle$ to scale the PDF of cloud mass fluxes:

$$\frac{\langle M \rangle}{\langle m \rangle^2} e^{-m/\langle m \rangle} \frac{\delta t}{t_L} \delta m.$$

The extra factor $\delta t/t_L$ is to account for the finite lifetime $t_L$ of the clouds ($\delta t$ is the model time step).

- Split this PDF into bins $\delta m$ and choose randomly (based on the probability for that bin) whether or not to initiate a cloud for each bin.

- For each cloud initiated, use the Kain-Fritsch plume model to determine vertical tendency profiles.

- These tendency profiles last for a time $t_L$. Clouds older than $t_L$ are removed and the remaining tendency profiles are summed to give the overall tendency profiles to feed back to the dynamics.
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German Icosahedral Nonhydrostatic General Circulation Model (ICON)

- Jointly developed by DWD and the MPIM in Hamburg.
- The results shown here are for different constant resolutions (40 km to 160 km), to determine how the rainfall variability varies with resolution.
- Input averaging (equivalent length 80 km) is applied for the 40 km run; the effects of varying this averaging are also shown.
- The Aquaplanet setup is used here in order to isolate variability due to different schemes.
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PDFs of 6-hour rainfall accumulation for the two different deterministic schemes, at three different resolutions.

- Every 6 hours for 3–6 months, between ±20 degrees latitude.
PDFs of 6-hour rainfall accumulation for the stochastic scheme, at three different resolutions.

- Every 6 hours for 3–6 months, between ±20 degrees latitude.
PDFs of 6-hour rainfall accumulation for the stochastic scheme, upscaled onto the 160 km grid.

- Every 6 hours for 3–6 months, between ±20 degrees latitude.
PDFs of 6-hour rainfall accumulation for the stochastic scheme, upscaled, with no input averaging.

- Every 6 hours for 3–6 months, between ±20 degrees latitude.
PDFs of instantaneous mass flux ($\langle M \rangle$ from the CAPE closure) for the stochastic scheme

- Every 6 hours for 10–50 days, between $\pm 20$ degrees latitude.
- Results are for 40 km grid spacing.
PDFs of instantaneous rainfall rate for the stochastic scheme, for different amounts of input averaging.

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PDFs of 6-hour rainfall accumulation for the stochastic scheme, for different amounts of input averaging.

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Effect on distribution of temperature at 2.67 km

![Graph showing the distribution of temperature at 2.67 km](image)

- **PDF (K⁻¹)**
- **Temperature (K)**
- **Deterministic**
- **Stochastic**
The setup described here is what was used in this study.

- 24 members, 24 km grid length.
- Domain over Europe and the North Atlantic.
- 2 forecasts per day, 10–31 July 2009 (34 forecasts in total); 54 hour lead time.
- MOGREPS forecasts with the Plant-Craig scheme ("EXP") are verified in comparison with the Gregory-Rowntree scheme ("CTL").
- Rainfall over the UK is investigated in detail using NIMROD data.
Met Office (Global and) Regional Ensemble Prediction System

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Mean rainfall accumulation

12–18 hour forecast

48–54 hour forecast

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Rainfall accumulation variability

12–18 hour forecast

48–54 hour forecast

24 km

120 km
Model spread and error

Spread / Error (mm in 6 hours)

Forecast horizon (hours)

error GR
error PC
spread GR
spread PC

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Brier Skill Scores

Threshold: 0.3 mm

Threshold: 3.0 mm

24 km

120 km

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Convective equilibrium experiment

- UK Met Office UM used in idealised mode.
- Square domain, with bicyclic boundary conditions.
- 32km grid length, 512km total domain size.
- Constant sea-surface temperature applied; the surface heat transfer is allowed to vary.
- The atmosphere is forced by a uniform imposed cooling profile.
- The ensemble mean total mass flux $\langle M \rangle$ is constant.
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Convective equilibrium experiment – Cohen-Craig theory

Given an average number of clouds $\langle N \rangle$, the actual number of clouds $N$ follows a Poisson distribution:

$$p_{\langle N \rangle}(N) = \frac{\langle N \rangle^N e^{-\langle N \rangle}}{N!}.$$  

Combining this with the probability that the mass flux is $M$, given $N$:

$$p_N(M) = \int_0^M p_{N-1}(M-u)p(u)du$$

leads to a PDF of total mass flux:

$$p(M, \langle m \rangle, \langle M \rangle) = \delta(M)e^{-\langle M \rangle} + \frac{1}{\langle m \rangle} \sqrt{\frac{\langle M \rangle}{M}} e^{-\frac{M+\langle M \rangle}{\langle m \rangle}} l_1 \left( \frac{2}{\langle m \rangle} \sqrt{M\langle M \rangle} \right).$$
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Comparing rainfall PDFs

- Assume that convective rainfall $C$ is a linear function of mass flux $M$.
- Then a PDF of rainfall $p(C, \langle c \rangle, \langle C \rangle)$ can be derived, with the same shape as $p(M, \langle m \rangle, \langle M \rangle)$.
- This allows a comparison of PDFs from different schemes.
- This can be done for different scales, by looking at the rainfall over different numbers of grid boxes.
- Because $\langle c \rangle$ is not known, it is fitted to give the best agreement, for one scale.
  - This same value is then used for all scales.
  - In this way, the scale adaptivity of each scheme can be assessed.
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Rainfall PDFs for Gregory-Rowntree scheme

Rainfall rate (kgm\(^{-2}\) s\(^{-1}\))

Probability

Data

Theory

256 km

64 km

128 km

32 km

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Rainfall PDFs for Kain-Fritsch scheme

256 km

64 km

128 km

32 km
Rainfall PDFs for Plant-Craig scheme

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References

- Keane RJ, Plant RS, Tennant WJ. The Plant-Craig stochastic convection scheme in MOGREPS. In preparation.
- Keane RJ, Plant RS. 2012. Large-scale length and time-scales for use with stochastic convective parametrization. Q. J. R. Meteorological Soc. DOI:10.1002/qj.992